

INSTITUTE OF ECONOMIC STUDIES

Faculty of social sciences of Charles University

Swaps

Lecturer's Notes No. 4

Course: Financial Market Instruments II

Teacher: Oldřich Dědek

VII. INTEREST RATE SWAPS

7.1 The swap mechanism

interest rate swap is a contract which commits two counterparties to exchange over an agreed period two streams of interest payments, each calculated using a different interest rate

interest payments are applied to a common notional principal amount (there is no exchange of principal, this figure is used only to calculate the interest amounts to be exchanged)

interest payments are netted thereby reducing credit risk

swap is a derivative financial instrument because it makes payments that are derived from a cash instrument but does not employ this cash instrument to fund the payments

swap is an off-balance sheet instrument because it does not impact on the balance sheets of the swap counterparties but only on their profit and loss accounts

termination of a swap is the cancellation of the swap contract (in which case one counterparty compensates the other counterparty for the loss of expected profit over the remainder of the life of the swap)

assignment of a swap is the sale of the swap by one of the counterparties to a third party with an agreement of the other counterparty

coupon swap is fixed-against-floating swap which involves the exchange of an interest stream based on a fixed interest rate for an interest stream based on a floating interest rate

fixed interest rate is fixed over the life of the swap so the stream of interest paid consists of equal interest amounts each calculated at known rate in advance

floating interest rate applies to a given interest period and once this period expires a new rate must be fixed (reset) for the next interest period



Assume a five-year US dollar coupon swap which involves the exchange of a fixed rate of 7.5 % paid annually for 3-month LIBOR. Interest payments are calculated on a notional amount of 200 mil USD.

There would be 5 fixed interest payments paid at the end of each one-year period:

$$\text{fixed interest} = 200\,000\,000 \times 0.075 = 15\,000\,000$$

There would be 20 different floating interest payments paid at the end of each 3-month period:

$$\text{floating payment} = 200\,000\,000 \times \text{LIBOR}_t \times \frac{1}{4}$$

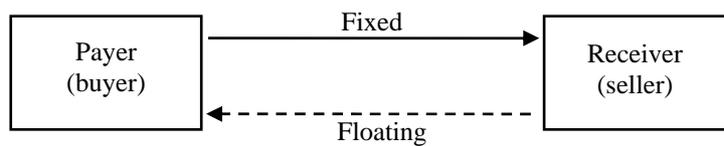


counterparties to coupon swap

payer and receiver in a swap refer to the fixed interest rate

buyer of a swap refers to the obligation to pay fixed interest rate

seller of a swap refers to the obligation to receive fixed interest rate



basis swap is floating-against-floating swap which involves a variety of combinations of floating interest rates

different tenors of the same interest rate (i.e. 3-month LIBOR against 6-month LIBOR)

the same or different tenors of the different interest rates (i.e. 3-month LIBOR against 3-month Treasury)

the same tenor of the same interest rate but with one carrying a margin (i.e. 3-month LIBOR against 3-month LIBOR plus 50 basis points)

counterparties to coupon swap

ambiguous convention to call parties payer-receiver or buyer-seller

each counterparty should be described in terms of the interest stream it pays or receives

generic swap (straight swap, plain vanilla swap) describes the simplest construction of a swap contract (constant notional amount, constant fixed rate, flat floating rate with no margins, regular interest payments, immediate start, no special risk features)

asset swap is a combination of a swap with an investment into a specific asset whose aim is to create a synthetic asset

liability swap is a combination of a swap with a commitment to honour a specific liability whose aim is to create a synthetic liability

term swap is a swap with an original tenor of more than two years

money market swap is a swap with an original tenor of up to two years

quotation conventions:

	two-way prices	spread over benchmark
2 year	5.70-5.75	21/25
3 year	6.23-6.28	40/45
5 year	7.01-7.05	46/51

two-way prices (bid-ask):

a higher-lower quote means that the quoting dealer is willing to transact a swap in which he receives/pays the given fixed rate

the difference between the two rates is the dealing spread the dealer earns on every matching pair of swaps

swap rate is the average of bid and ask interest rates

swap spread:

a higher/lower quote means that the quoting dealer is willing to transact a swap in which he receives/pays the given spread over the yield on a benchmark security

the benchmark interest rate is usually the yield on the most liquid government bond with remaining maturity closest to that of the swap

7.2 Speculative strategies with interest rate swaps

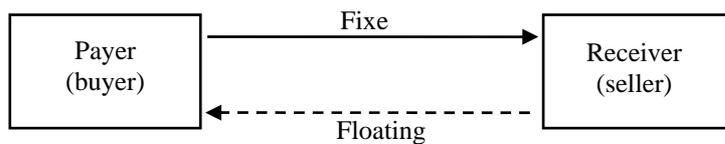
interest rate swaps can be used to take risk positions based upon expectations about the direction in which interest rates will move in the future

types of speculation strategies:

- taking risk positions independently of any underlying instrument
- transforming risk exposure to an individual underlying instrument or the whole balance sheet (asset and liability swaps are used for speculative purposes)

i) taking interest rate risk

- a) using coupon swap



the buyer of a coupon swap may expect that the differential between the two interest rates will change such that the floating interest rate will rise above the fixed rate

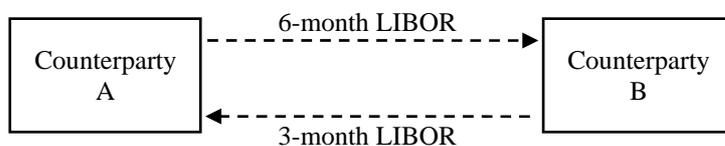
the seller of a coupon swap may expect that the differential between the two interest rates will change such that the floating interest rate will fall below the fixed rate

analogy with asset-liability mismatching (*gapping*):

the payer of the swap issues a bond (then pays a fixed coupon) and rolls over a short term deposit (then receives floating interest)

similarly for the buyer of the coupon swap

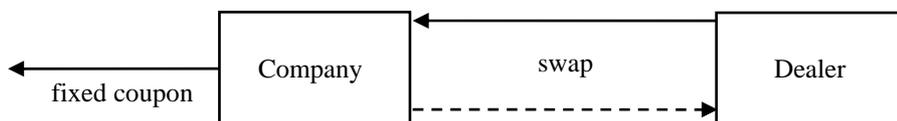
b) using basis swap



counterparties in a basis swap may expect that the differential between the two interest rates will change such that the net effect will increase the overall profitability of the swap

ii) transforming interest rate risk of an individual instrument

a) transformation of fixed-interest liability to floating-interest liability



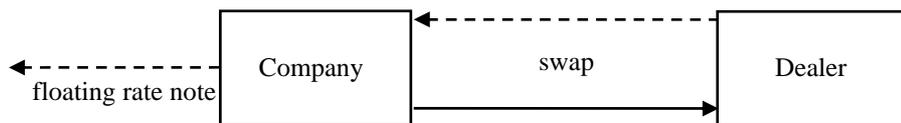
a company has issued a fixed-income bond and is thus exposed to the risk of a fall in interest rates (it will pay a higher rate of interest on its borrowing than necessary)

if the company wishes to benefit from an expected fall in interest rates it may put on a coupon swap in which it receives fixed and pays floating interest

the company would still have the fixed-income bond on its balance sheet but the swap has changed the cash flow characteristics of this liability (a synthetic floating-interest liability was created)

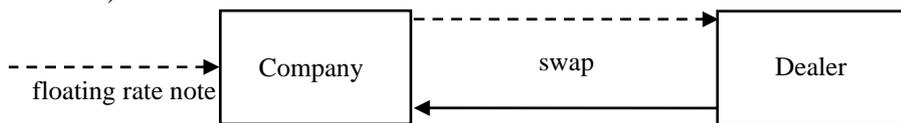
b) transformation of floating-interest liability to fixed-interest liability

creation of a synthetic fixed-interest liability (company anticipates a rise in interest rates)



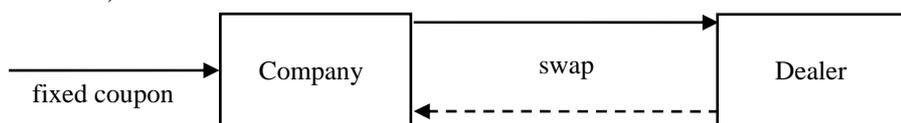
c) transformation of floating-interest asset to fixed-interest asset

creation of a synthetic fixed-interest asset (company anticipates a fall in interest rates)



d) transformation of fixed-interest asset to floating-interest asset

creation of a synthetic floating-interest asset (company anticipates a rise in interest rates)

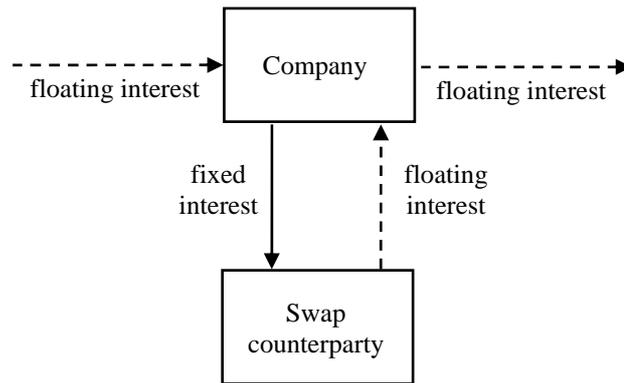


iii) changing interest rate risk of the balance sheet

transformation of a hedged position into speculation on an interest rate rise

a bank initially has no exposure to interest rate risk because both assets and liabilities derive their cash flows from floating rates

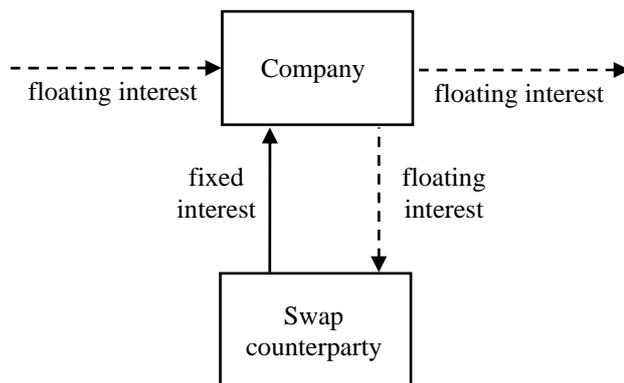
if the bank wishes to take a risk position based on an expected interest rate rise it could do so by putting on a coupon swap in which it is the payer of fixed and receiver of floating interest



transformation of a hedged position into speculation on an interest rate fall

a bank initially has no exposure to interest rate risk because both assets and liabilities derive their cash flows from floating rates

if the bank wishes to take a risk position based on an expected interest rate fall it could do so by putting on a coupon swap in which it is the receiver of fixed and payer of floating interest



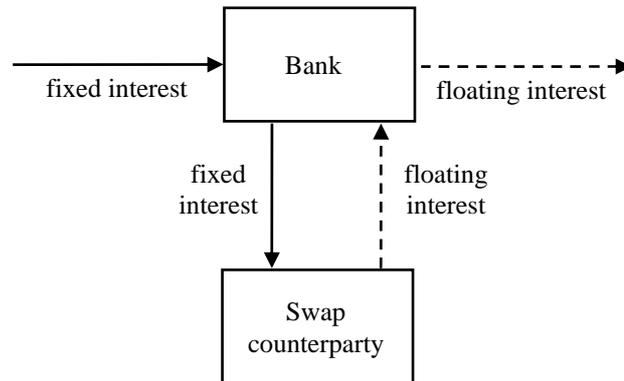
similar charts can be developed for the case when both assets and liabilities derive their cash flows from fixed interest rates

7.3 Hedging strategies with interest rate swaps

a hedge is a risk taken to offset an equal and opposite risk

swaps can provide the necessary offsetting risk

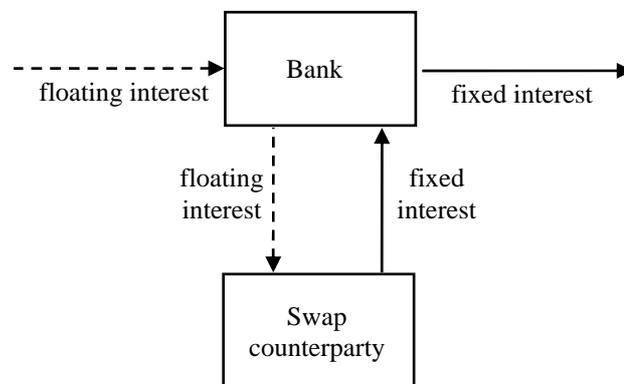
i) hedging with coupon swaps



a bank is exposed to the risk that interest rates may rise (the cost of funding will be increased without offsetting benefits of any increase in the returns on its assets)

a coupon swap accomplishes that cash flows to and from the swap offset cash flows from and to the bank's balance sheet

the hedging scheme is a typical way mortgage lenders can fund fixed-rate mortgages from floating-rate deposits while avoiding exposure to interest rate risk



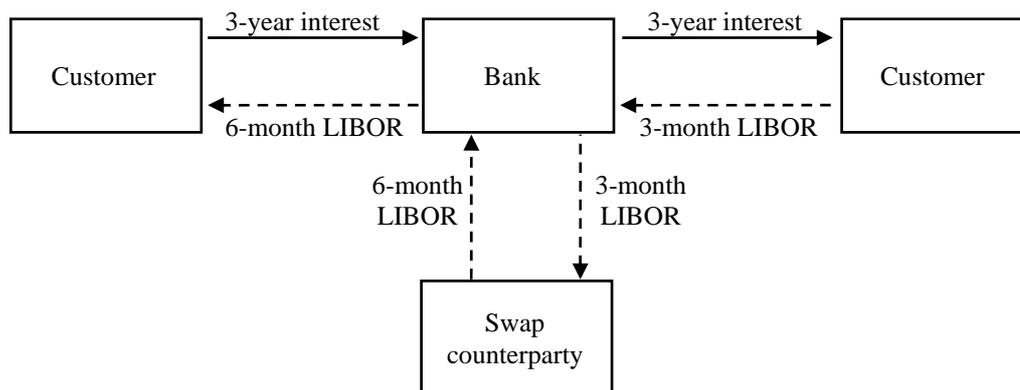
a bank is exposed to the risk that interest rates may fall

the hedging scheme is a typical way mortgage lenders can fund floating-rate mortgages from fixed rate liabilities such as bonds and longer-term certificate of deposits

ii) hedging with basis swaps

a bank transacts two coupon swaps for different customers and is left with a basis risk between 3-month and 6-month LIBOR

a basis swap can hedge the risk between the two indexes



7.4 Arbitrage strategies with interest rate swaps

an arbitrage opportunity exists if one instrument generates a higher rate of interest than another but they both calculate interest using the same index
 in efficient markets arbitrage opportunities should not exist but in practice price discrepancies occur and give rise to arbitrage opportunities
 by exchanging interest payments based on different indexes swaps play a crucial role in the integration and globalization of the financial markets

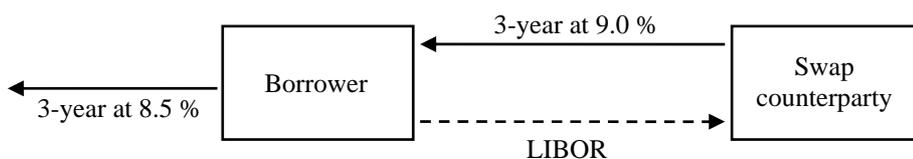
i) arbitraging liabilities

a swap can reduce the cost of preferred form of funding if a swap's user has access to a cheaper form of an available but non-preferred cost of funding
 reasons for interest rate advantage

- swap rates reflect standard market yields and are not adjusted to take account of variations in the creditworthiness of swap counterparties
- subsidized finance
- different speed at which different financial markets respond to the same information
- short-term price anomalies



A firm has access to a fixed interest rate of 8.5 % which is cheaper relative to prevailing market rates. The firm can make a turn by putting on a swap in which it receives a higher market fixed rate of 9.0 %. The turn of 0.5 % can be used to subsidize the floating rate paid out creating a synthetic floating rate funding of LIBOR – 0.5 %



ii) arbitraging assets

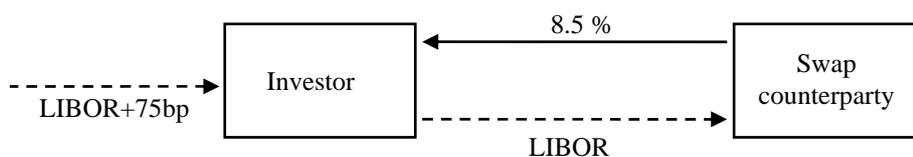
a swap can enhance return on a preferred form of investment if a swap's user has access to an asset whose yields is above prevailing market rates

reasons for interest rate advantage

- assets are illiquid or difficult to price and therefore have to pay abnormally higher yields in order to compensate investors for the risks involved (complex securities targeted at narrow groups of investors, illiquid bond issues, etc.)
- variations in the speed at which different financial markets respond to the same information



An investor holds a FRN (floating rate note) with a yield of 75 basis points above LIBOR while the floating rate in swaps is normally flat LIBOR. The investor can make a turn by putting on a swap in which it pays flat LIBOR. The turn of 0.75 % can be used to supplement the fixed interest received through the swap creating a synthetic fixed rate asset with an enhanced yield



iii) new issue arbitrage

new issue arbitrage is a strategy that exploits arbitrage opportunities that arise because of differences between the credit risk premiums demanded from the same counterparty by cash and swap markets



Two companies, one with an AA credit rating and the other with an A rating, can raise funds in the fixed-income bond market or through floating-rate bank loans at the following terms:

	Fixed rate	Floating rate
Company AA	10 %	LIBOR + 100 bp
Company A	12 %	LIBOR + 160 bp
Differential	200 bp	60 bp
Arbitrage potential = 200 – 60 = 140 bp		

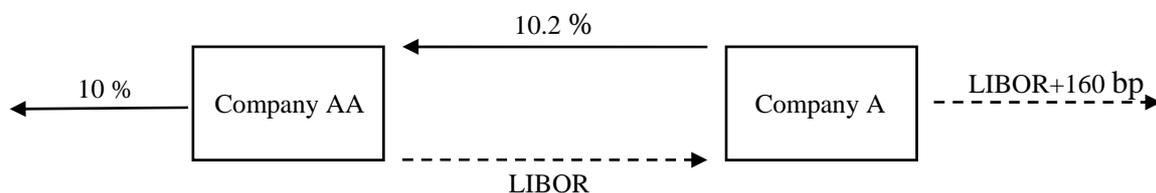
the firm AA can fund itself more cheaply in both markets \Rightarrow AA has an *absolute advantage* in both markets and A has absolute disadvantage in both markets

the AA's interest rate advantage is greater in the fixed-rate market \Rightarrow AA has a *relative advantage* in the fixed-rate market and *relative disadvantage* in the floating-rate market

the A's interest rate disadvantage is less in the floating-rate market \Rightarrow A has a *relative advantage* in the floating-rate market and relative disadvantage in the fixed-rate market

both firms can gain from putting on an interest rate swap if they want to end up with interest rate borrowing in the markets where they have comparative disadvantage (AA wants to pay floating rate and A wants to pay fixed rate)

arrangement of a swap: both firms borrow funds in the markets where they have comparative advantage and then swap interest payments in such a way that both save borrowing cost relative to those achieved without the swap



net cost of funds to AA:

fixed interest on bonds	– 10 %
floating interest through swap	– LIBOR
fixed interest through swap	+ 10.2 %
net cost	– (LIBOR – 20 bp)

company AA reduced the cost of its floating-rate funding by 120 basis points

net cost of funds to A:

floating interest on bank loan	– (LIBOR + 160 bp)
fixed interest through swap	– 10.2 %
floating interest through swap	+ LIBOR
net cost	– 11.8 %

company A reduced the cost of its fixed-rate funding by 20 basis points

the two companies save together $120 + 20 = 140$ basis points

the arbitrage potential of 140 basis points has been distributed between the two firms in a proportion of 120 and 20 basis points

notes

i) the mechanism of the new issue arbitrage is similar to that proposed by David Ricardo in his explanation of international trade (famous theory of comparative advantage)
the less efficient country should specialize in its least inefficient line of production
the more efficient country should specialize in its most efficient line of production

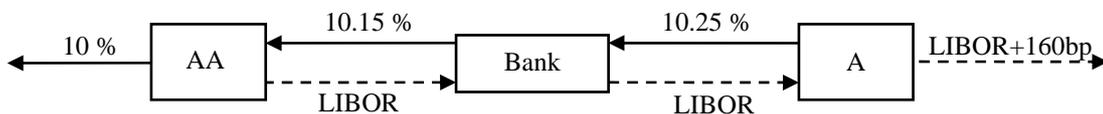
ii) the strategy may be used to gain access to bond market which would not otherwise be available to borrowers

a company should have a target figure for its net cost of funds

iii) the structure is likely to be complicated by the intermediary bank that acts as a swap dealer

corporate counterparties are traditionally reluctant to take on the credit risk of other business firms

the intermediary bank shares a part of the arbitrage potential



distribution of the arbitrage potential

115 basis points for the company AA

15 basis points for the company A

10 basis points for the bank

iv) the evaluation of gains from a new issue arbitrage is based on unchanged credit rating of swap counterparties during the swap



Suppose that the company A was downgraded just after the above swap transaction had been arranged and thereby its borrowing spread over LIBOR changed from 160 to 200 basis points. Its new net cost of funding at the fixed rate is thus

$$\text{LIBOR} - (\text{LIBOR} + 200 \text{ bp}) - 10.2 \% = -12.2 \%$$

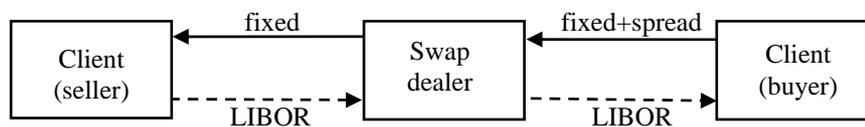
It is more by 20 basis points in comparison with direct borrowing at the fixed interest rate of 12 %.



7.5 Warehousing the interest rate swap

warehousing is a temporary hedging of interest rate risk generated by the swap contract until a matching or reverse swap can be agreed

hedged position



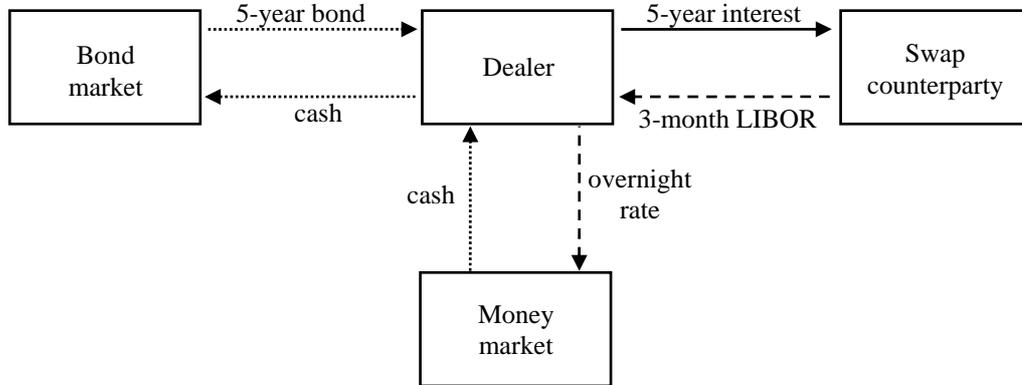
until the dealer finds a matching swap he is exposed to the interest rate risk

if he is the payer of a fixed rate he is exposed to the interest rate fall that would reduce the fixed rate on new swaps (a matching swap will pay less fixed interest than he is paying out through the swap)

if he is the receiver of a fixed rate he is exposed to the interest rate rise that would increase the fixed rate on new swaps (a matching swap will pay a higher fixed interest than he is receiving through the swap)

a) warehousing with bonds

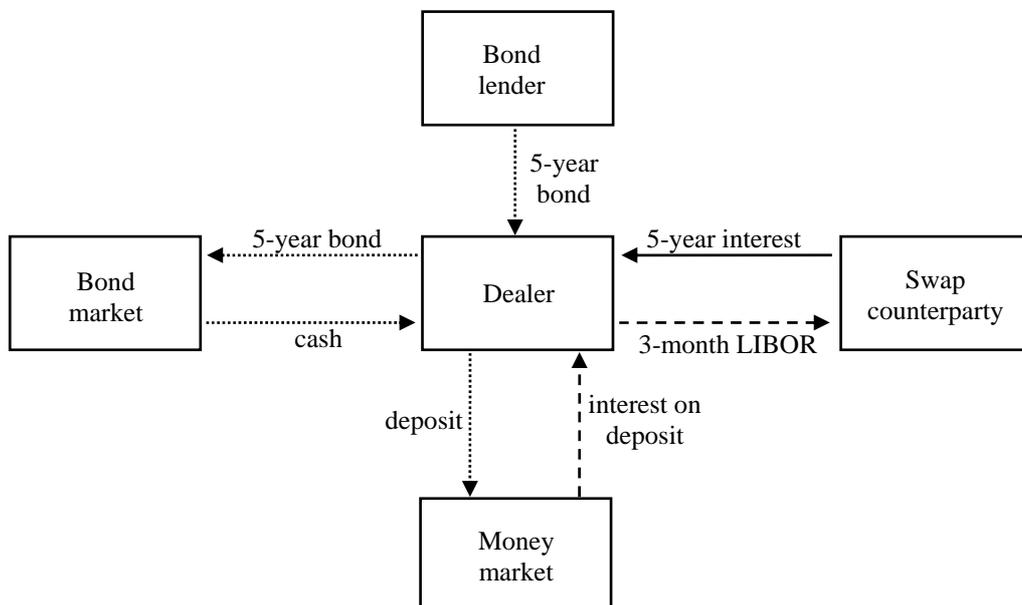
i) dealer is the payer of a fixed rate and is concerned about an interest rate fall



steps in warehousing the temporarily unhedged swap

- the dealer arranges a rolled-over overnight borrowing whose proceeds are used to buy bonds with the same maturity as the swap
- in case of an interest rate fall the capital gain on bonds should more or less offset the income loss on the swap
- the cost of funding is partially offset by the floating interest received through the swap (dealer is exposed to some basis risk because overnight rates are repriced each day whereas the floating rate is a 3-month LIBOR)
- alternatively the purchase of bonds could be financed through a repo instead of direct borrowing in the money market

ii) dealer is the receiver of a fixed rate and is concerned about an interest rate increase

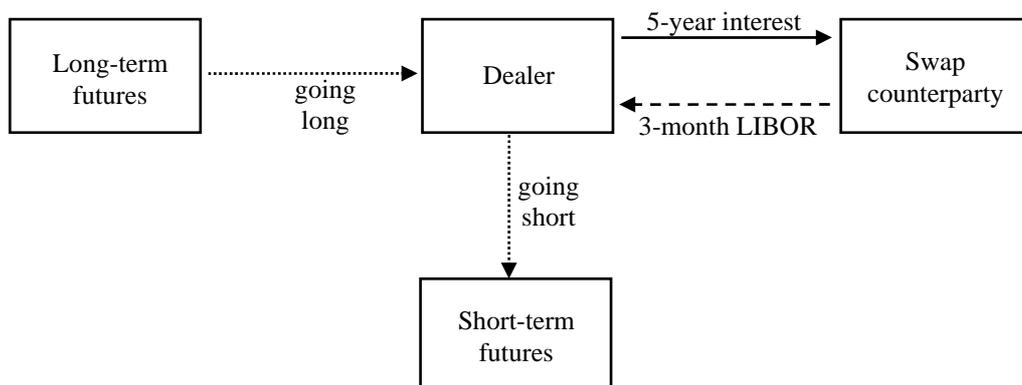


steps in warehousing the temporarily unhedged swap

- the dealer borrows appropriate bonds that are sold to establish a short position
- the proceeds from the sale are deposited in the money market and the interest received is used to pay the floating interest through the swap
- in case of an interest rate rise (and a fall in the bond price) the return from selling bonds short offsets the income loss on the swap
- alternatively the bonds could be borrowed through a reverse repo instead of short selling

a) warehousing with futures

- long position in futures is appropriate for hedging an expected interest rate fall and short position in futures is appropriate for hedging an expected interest rate rise
- the fixed interest stream in a coupon swap can be hedged with long-term interest rate futures contracts (futures on government bonds)
- the floating interest stream can be hedged with short-term interest rate futures contracts
- an advantage of warehousing with futures is that they require, similarly to swap contracts, no payments of principal
- a disadvantage of warehousing with futures is considerable basis risk in hedging fixed interest stream as the tenors of long-term interest rate futures are usually much longer than those of swap contracts



7.6 Valuation of swaps

valuation of a swap consists in the assessment of the net worth of the two future streams to be exchanged through the swap

value for the swap buyer = PV(floating stream) – PV(fixed stream)

value for the swap seller = PV(fixed stream) – PV(floating stream)

$$\text{PV(fixed stream)} = \sum_{t=1}^T \frac{cM}{(1+z_t)^t} = cM \frac{1-(1+r_T)^{-T}}{r_T}$$

$$\text{PV(floating stream)} = \sum_{t=1}^T \frac{f_t M}{(1+z_t)^t} = M - \frac{M}{(1+z_T)^T}$$

M ... principal notional amount of the swap

c ... fixed interest paid in the swap

z_t ... discount factor at time t (YTM of t -year zero-coupon bond)

r_T ... swap rate for a given maturity (YTM of T -year coupon bond)



What is the value of the coupon swap which has a notional principal amount of 55 mil USD, pays semi-annually a fixed interest of 10.63 % and has 4.5 years remaining to maturity? Assume the current swap rate for 4.5-year swap is 10.12%. Suppose a flat yield curve.

The assumption of a flat yield curve implies that the swap rate 10.12 % can be used as a discount rate for calculating the PVs of both legs of the swap ($r_T = z_T = 10.12\%$).

$$\text{PV(fixed stream)} = \frac{1}{2} \times 0.1063 \times 55000000 \times \frac{1 - (1 + \frac{1}{2} \times 0.1012)^{-9}}{\frac{1}{2} \times 0.1012} = 20722536$$

$$\text{PV(floating stream)} = 55000000 - \frac{55000000}{(1 + 0.1012)^{4.5}} = 19357777$$

The value of the swap (to the counterparty receiving fixed interest) is therefore

$$\text{NPV} = 20722536 - 19357777 = 1364759$$



pricing of the swap is the setting of the fixed interest rate for new coupon swaps of the setting of margins between floating interest rates for new basis swaps
the value of a generic interest rate swap priced at current market rates should be zero because neither counterparty should rationally agree to a contract in which they expect to make a loss (the NPV of their payments exceeds the NPV of their receipts)

$$\text{NPV}(\text{swap}) = 0 \Rightarrow \text{PV}(\text{fixed stream}) = \text{PV}(\text{floating stream})$$

the condition of zero NPV is used for setting the fixed interest rate in new coupon swaps

VIII. FORWARD RATE AGREEMENT

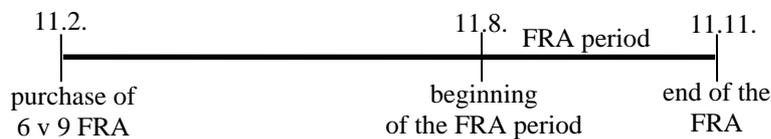
8.1 Basic concepts

forward rate agreement (FRA) is an agreement to pay or receive, on an agreed future date, the difference between an agreed rate (called *FRA rate*) and the rate actually prevailing on that future date (called *reference rate*)

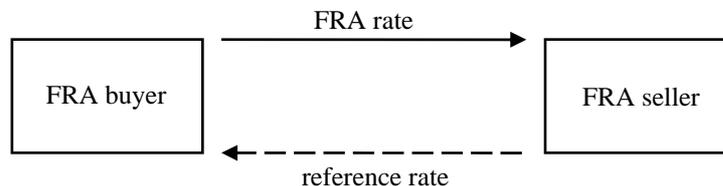
market conventions:

- FRA is referred to by the beginning and end dates of the covered period which is called FRA period (notation is 6 versus 12, 6 v 12, 6 * 12)

6 v 9 FRA



- FRA buyer is a party which pays a predetermined FRA rate and obtains the reference rate while FRA seller is the other party which pays the reference rate and obtains the predetermined FRA rate



- the settlement cash flows in a FRA contract are calculated on an agreed notional principal amount and are paid as a net rather than a gross amount
- the settlement conventionally takes place at the beginning of the FRA period (it is therefore discounted to a present value at the current reference rate)
- interest payments are calculated using the simple interest (in accordance with money market conventions)

cash flow for the FRA buyer:

$$CF = M \times \frac{({}_t r_p - {}_t k_p) \times \frac{p-t}{360}}{1 + r_t \times \frac{p-t}{360}}$$

M ... notional amount

$p - t$.. FRA period starting at a time t and ending at a time p (number of days)

${}_t r_p$... reference rate prevailing at the beginning of the FRA period

${}_t k_p$... fixed FRA rate determined when the contract is negotiated

cash flow for the FRA seller:

$$CF = M \times \frac{({}_t k_p - {}_t r_p) \times \frac{p-t}{365}}{1 + {}_t r_p \times \frac{p-t}{365}}$$



On 11 February a customer buys a FRA contract 6 v 9 at a FRA rate of 12.8 %. The reference rate is the 3-month EURIBOR. The contract is based on a notional amount of 1 million EUR. Suppose that the actual 3-month EURIBOR at the beginning of the FRA period (that is on 11 August) is 11.5 %. Net cash flow for the FRA buyer is

$$1000000 \times \frac{(0.115 - 0.128) \times \frac{92}{365}}{1 + 0.115 \times \frac{92}{365}} = -3184.4$$

On 11 August thus the customer pays the dealer 3184.4 EUR.

Alternatively suppose that the actual 3-month EURIBOR at the beginning of the FRA period is 13.5 %. Net cash flow for the FRA buyer is

$$1000000 \times \frac{(0.135 - 0.128) \times \frac{92}{365}}{1 + 0.135 \times \frac{92}{365}} = 1706.3$$

On 11 August thus the customer receives from the dealer 1706.3 EUR.



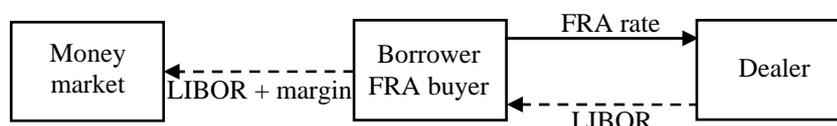
8.2 Applications of FRA

a) hedging using FRA contracts

FRA contracts are suitable instruments for locking in short-term interest rates



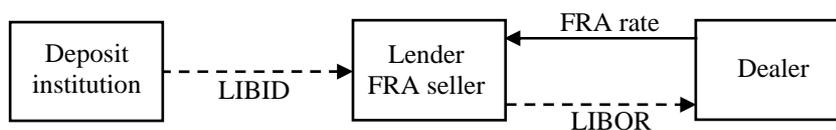
A company rolls over every three months a bank credit in which the interest rate is refixed at the prevailing 3-month LIBOR plus a given margin. The company expects interest rates to rise before the next rollover date which is 2 months away. It therefore buys a “2 v 5 FRA”. In this way the company’s borrowing cost will be locked in.



company pays	LIBOR + margin	to lending bank
company pays	FRA rate	to FRA counterparty
company receives	LIBOR	from FRA counterparty
<hr/>		
net cost	FRA rate + margin	



A company expects to make a 6-month deposit in two weeks' time and fears that interest rate may fall. The company therefore sells a "2-week v 6½" FRA. In this way the company will lock in the deposit rate.



company receives	LIBID	from deposit
company receives	FRA rate	from FRA counterparty
company pays	LIBOR	to FRA counterparty
<hr/>		
net return	FRA rate – (LIBOR – LIBID)	



b) speculation using FRA contracts

FRA can be used to speculate on whether the cash interest rate prevailing when the FRA period begins is higher or lower than the FRA rate.

if the trader expects interest rates to rise he buys an FRA

if the trader expects interest rates to fall he sells an FRA

8.3 FRA strips

FRA strip is a series of consecutive FRAs which compounded together build up to a longer term FRA contract

examples: - the series of FRAs (3 v 6) & (6 v 9) builds up to a 3 v 9 FRA

- 2-month LIBOR (seen as 0 v 2 FRA rate) plus the series of FRAs (2 v 6) & (6 v 12) builds up to 1-year rate (seen as 0 v 12 FRA rate)

interest rate parity for a FRA strip

$$\left(1 + k_1 \times \frac{d_1}{365}\right) \times \left(1 + k_2 \times \frac{d_2}{365}\right) \times \dots \times \left(1 + k_n \times \frac{d_n}{365}\right) = \left(1 + k_s \times \frac{d_1 + \dots + d_n}{365}\right)$$

k_1, \dots, k_n ... FRA rates of consecutive FRA contracts

d_1, \dots, d_n ... FRA periods of consecutive FRA contracts

k_s ... FRA rate of compounded FRA



Suppose that following rates are available:

3-month LIBOR	8.2 % (92 days)
3 v 6 FRA	8.7 % (91 days)
6 v 9 FRA	9.5 % (91 days)

A strip 3M & (3 v 6) & (6 v 9) can be constructed that builds up to a 9-month borrowing rate.

No arbitrage condition requires that the 9-month borrowing rate is

$$r_9 = \left[\left(1 + 0.082 \times \frac{92}{365}\right) \times \left(1 + 0.087 \times \frac{91}{365}\right) \times \left(1 + 0.095 \times \frac{91}{365}\right) - 1 \right] \times \frac{365}{274} = 9.0 \%$$

Using available financial instruments the 9-month borrowing can be created synthetically as follows:

Now:

Borrowing cash for 3 months (cash inflow = +1)

Buying 3 v 6 FRA (notional amount = 1.02067) ←

Buying 6 v 9 FRA (notional amount = 1.04281) ←

After 3 months

Repayment of 3M loan (cash outflow = $-(1 + \text{LIBOR}_0) = -(1 + 0.082 \times \frac{92}{365}) = -1.02067$)

Settlement of 3 v 6 FRA (cash inflow = $1.02067 \times (\text{LIBOR}_3 - 0.087 \times \frac{91}{365}) / (1 + \text{LIBOR}_3)$)

Refinancing the balance by taking a new 3M loan

computing the balance =

$$-1.02067 + 1.02067 \times \frac{\text{LIBOR}_3 - 0.087 \times \frac{91}{365}}{1 + \text{LIBOR}_3} = -1.02067 \times \frac{1 + 0.087 \times \frac{91}{365}}{1 + \text{LIBOR}_3} = -\frac{1.04281}{1 + \text{LIBOR}_3}$$

new 3-month loan (cash inflow = $+1.04281 / (1 + \text{LIBOR}_3)$)

After 6 months

Repayment of loan (cash outflow = $-[1.04281 / (1 + \text{LIBOR}_3)] \times (1 + \text{LIBOR}_3) = -1.04281$)

Settlement of 6 v 9 FRA (cash inflow = $1.04281 \times (\text{LIBOR}_6 - 0.095 \times \frac{91}{365}) / (1 + \text{LIBOR}_6)$)

Refinancing the balance by taking a new 3M loan

computing the balance =

$$-1.04281 + 1.04281 \times \frac{\text{LIBOR}_6 - 0.095 \times \frac{91}{365}}{1 + \text{LIBOR}_6} = -1.04281 \times \frac{1 + 0.095 \times \frac{91}{365}}{1 + \text{LIBOR}_6} = -\frac{1.06751}{1 + \text{LIBOR}_6}$$

new 3-month loan (cash inflow = $+1.06751 / (1 + \text{LIBOR}_6)$)

After 9 months

Repayment of loan (cash outflow = $-[1.06751 / (1 + \text{LIBOR}_6)] \times (1 + \text{LIBOR}_6) = -1.06751$)

Effective 9-month borrowing rate

$$1 + r_{9M} \times \frac{274}{365} = 1.06751 \Rightarrow r_{9M} = 0.06751 \times \frac{365}{274} = 9.0 \%$$



8.4 Pricing links between FRAs and futures contracts

FRA is a negotiable equivalent to an interest rate futures contract (all specifications of

FRAs are entirely flexible)

net interest rate gain for the buyer of an interest rate futures contract (marking to market)

$$\text{gain} = F_T - F_0 = (100 - F_0) - (100 - F_T) = r_0 - r_T$$

r_0 ... an interest rate implied by the opening price of futures contract; it is known when the contract starts

r_T ... an interest rate implied by closing price of futures contract; it is determined later at the delivery date

net interest rate gain for the buyer of an FRA contract

$$\text{gain} = {}_t r_p - {}_t k_p$$

${}_t k_p$... FRA rate; it is known when the contract is negotiated

${}_t r_p$... reference rate; it is determined later when the contract is settled (at the beginning of the FRA period)

in both cases a net interest rate gain is the difference between two interest rates (the one known from the very beginning, the other determined later)

opposite conventions: a buyer of futures will profit if interest rates fall while a buyer of FRA will profit if interest rates rise

a) pricing standardized FRAs

standardized FRA is called the FRA contract which is priced at an interest rate implied by an interest rate futures contract

FRA rate should equal implied futures rate when

- the start of the FRA period coincides with the delivery date of the futures contract
- the FRA period coincides with the maturity of the underlying asset in the futures contract

the two rates should be the same because both instruments provide equivalent pattern of interest cash flow (otherwise arbitrage opportunities would exist)



Suppose the following prices are quoted on March 18 for given maturities of the ST3 futures contract in which the underlying asset is a three-month LIBOR sterling deposit:

June futures (delivery 17 June)	91.75 (implied rate 8.25 %)
September futures (delivery 17 September)	91.50 (implied rate 8.50 %)
December futures (delivery 17 December)	91.25 (implied rate 8.75 %)

Which FRA contracts settling against the LIBOR are likely to be priced at the above implied interest rates?

The June contract covers a three-month period starting on 18 June and ending on 18 September. Therefore on 18 March the FRA 3 v 6 which also starts on 18 June and ends on 18 September should have an FRA rate of 8.25 %.

For the same reason, on 18 March the 6 v 9 FRA which starts on 18 September and ends on 18 December should have an FRA rate of 8.50 %.

For the same reason, on 18 March the 9 v 12 FRA which starts on 18 December and ends on 18 March should have an FRA rate of 8.75 %.



b) pricing standardized FRA strips

FRA strip is a series of consecutive FRAs which compounded together build up to a longer term FRA contract

standardized FRA strip is a strip created from standardized FRA contracts

standardized FRA strips should be priced at FRA rates consistent with the interest rate parity condition



With the same standardized FRAs as in the previous example what standardized FRA strips can be created and at which FRA rates should they be priced on 18 March?

Having (3 v 6), (6 v 9) and (9 v 12) FRAs, the following strips can be created:

- (3 v 6) & (6 v 9) \Rightarrow (3 v 9) (number of days 92 + 91 = 183)
- (6 v 9) & (9 v 12) \Rightarrow (6 v 12) (number of days 91 + 90 = 181)
- (3 v 6) & (6 v 9) & (9 v 12) \Rightarrow (3 v 12) (number of days 92 + 91 + 90 = 273)

Given FRA rates of standardized FRAs

$${}_3k_6 = 8.25\%, \quad {}_6k_9 = 8.50\%, \quad {}_9k_{12} = 8.75\%$$

then the interest rate parity condition implies

$${}_3k_9 = \left[\left(1 + 0.0825 \times \frac{92}{365} \right) \times \left(1 + 0.0850 \times \frac{91}{360} \right) - 1 \right] \times \frac{360}{183} = 0.0846 = 8.46\%$$

$${}_6k_{12} = \left[\left(1 + 0.0850 \times \frac{91}{365} \right) \times \left(1 + 0.0875 \times \frac{90}{360} \right) - 1 \right] \times \frac{360}{181} = 0.0872 = 8.72\%$$

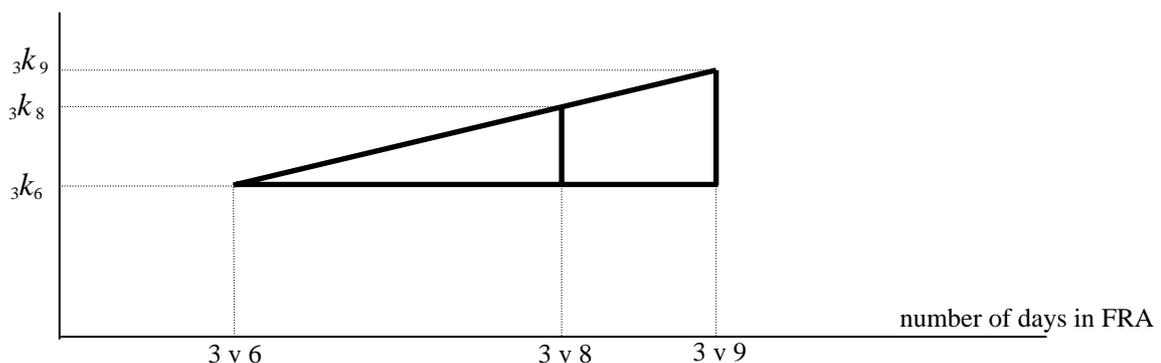


practical imperfections:

- the FRA rate of a standardized FRA strip assumes compounded notional amounts while futures contracts are for standardized notional amounts only
- different conventions in futures markets (ACT/360) and in FRA markets (ACT/365)
- rounded number of futures contracts
- slight discrepancies in dates for fixing the settlement LIBOR rates in futures and FRA contracts

c) pricing interpolated FRAs

1) the start of the FRA period coincides with a futures contract



for the 3 v 8 FRA we are effectively asking the following question:

what the market expects the 5-month rate to be in 3 months' time if the available information is what the market expects the 3-month and 6-month rates to be at the same time

property of similar triangles

$$\frac{{}_3k_9 - {}_3k_6}{\text{days in } (3 \text{ v } 9) - \text{days in } (3 \text{ v } 6)} = \frac{{}_3k_8 - {}_3k_6}{\text{days in } (3 \text{ v } 8) - \text{days in } (3 \text{ v } 6)}$$

solution for the 3 v 8 FRA

$${}_3k_8 = {}_3k_6 + ({}_3k_9 - {}_3k_6) \times \frac{\text{days in } (3 \text{ v } 8) - \text{days in } (3 \text{ v } 6)}{\text{days in } (3 \text{ v } 9) - \text{days in } (3 \text{ v } 6)}$$



Determine the FRA rate for the 3 v 8 FRA (5-month rate expected in 3 months' time) by using the method of linear interpolation. With the data from previous example we have

$${}_3k_6 = 8.25 \%, \quad {}_3k_9 = 8.46 \%,$$

$$\text{days in } (3 \text{ v } 6) = 92, \quad \text{days in } (3 \text{ v } 9) = 183, \quad \text{days in } (3 \text{ v } 8) = 153$$

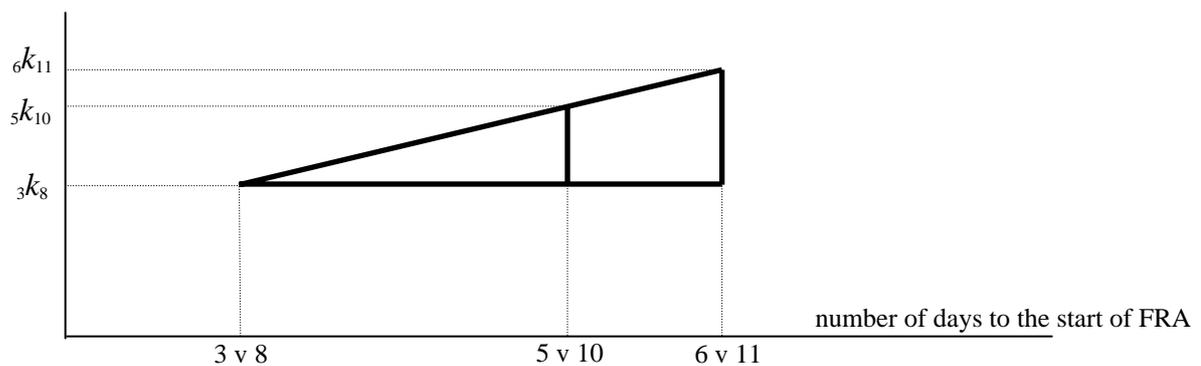
$${}_3k_8 = 0.0825 + (0.0846 - 0.0825) \times \frac{153 - 92}{183 - 92} = 0.0839 = 8.39 \%$$

For the same reason, on 18 March the (6 v 11) FRA (5-month rate expected in 6 months' time) can be interpolated in the following way:

$$\begin{aligned} {}_6k_{11} &= {}_6k_9 + ({}_6k_{12} - {}_6k_9) \times \frac{\text{days in } (6 \text{ v } 11) - \text{days in } (6 \text{ v } 9)}{\text{days in } (6 \text{ v } 12) - \text{days in } (6 \text{ v } 9)} \\ &= 0.085 + (0.0872 - 0.085) \times \frac{153 - 91}{181 - 91} = 0.0865 = 8.65 \% \end{aligned}$$



2. the start of the FRA period is between two futures contracts



for the 5 v 10 FRA we are effectively asking the following question:

what the market expects the 5-month rate to be in 5 months' time if the available information is what the market expects the 5-month rate to be in 3 months' time and 6 months' time

property of similar triangles

$$\frac{{}_6k_{11} - {}_3k_8}{\text{days to } (6 \text{ v } 11) - \text{days to } (3 \text{ v } 8)} = \frac{{}_5k_{10} - {}_3k_8}{\text{days to } (5 \text{ v } 10) - \text{days to } (3 \text{ v } 8)}$$

solution for the 5 v 10 FRA

$${}_5k_{10} = {}_3k_8 + ({}_6k_{11} - {}_3k_8) \times \frac{\text{days to } (5 \text{ v } 10) - \text{days to } (3 \text{ v } 8)}{\text{days to } (6 \text{ v } 11) - \text{days to } (3 \text{ v } 8)}$$



Determine the FRA rate for the 5 v 10 FRA (5-month rate expected in 5 months' time) by using the method of linear interpolation. With the data from previous examples we have

$${}_3k_8 = 8.39 \%, \quad {}_6k_{11} = 8.65 \%,$$

$$\text{days to } (3 \text{ v } 8) = 92, \text{ days to } (5 \text{ v } 10) = 153, \text{ days to } (6 \text{ v } 11) = 184$$

$${}_5k_{10} = 0.0839 + (0.0865 - 0.0839) \times \frac{153 - 92}{184 - 92} = 0.0856 = 8.56 \%$$



5. Price links between RFAs and interest rate swaps

both coupon swaps and FRA strips involve the exchange of the same types of interest cash flows (fix versus float)

no arbitrage condition creates price links between the two types of contracts



Suppose the following borrowing rates are available:

3-month LIBOR 14.1 % (91 days)

3 v 6 FRA 12.4 % (91 days)

6 v 9 FRA 11.6 % (91 days)

9 v 12 FRA 11.2 % (92 days)

What should be the rate for a 12-month interest rate swap against quarterly LIBOR payments?

One-year swap should be priced at a rate s which satisfies the equation:

$$1 + s = \left(1 + \frac{91}{365} \times 0.141\right) \times \left(1 + \frac{91}{365} \times 0.124\right) \times \left(1 + \frac{91}{365} \times 0.116\right) \times \left(1 + \frac{92}{365} \times 0.112\right) \\ = 1.129$$

One-year swap rate should be quoted at a price of 12.9 %.



IX. CURRENCY SWAPS

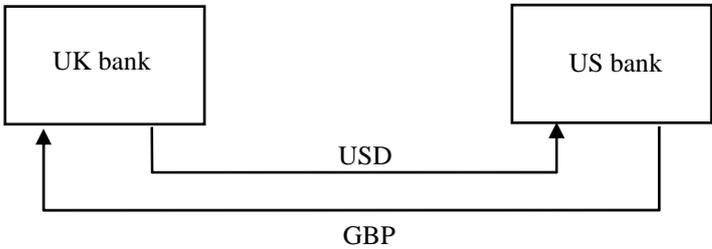
9.1 Related financial instruments

a) outright forward

outright forward is an agreement about the exchange of principal amounts denominated in two different currencies taking place at some future date

the future exchange will take place at the forward exchange rate (usually different from the spot rate)

there is no exchange of interest amounts but the differential between the spot and forward rates reflects differences in interest rates (in line with covered interest rate parity)

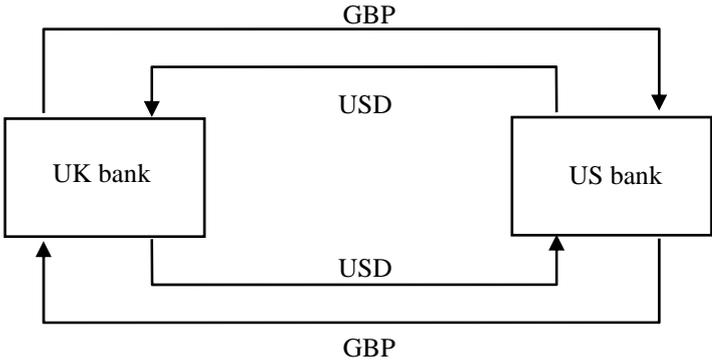


b) foreign exchange swap

foreign exchange swap is an agreement about the exchange of principal amounts denominated in two different currencies both at the start and at maturity of the swap

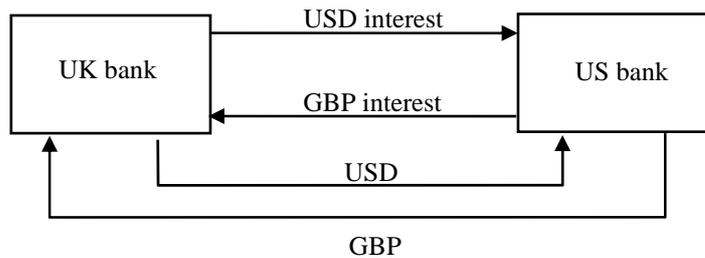
the initial exchange is made at the prevailing spot exchange rate and the re-exchange is made at the forward exchange rate prevailing at the start of the swap (because spot and forward exchange rates are usually different, the amounts of principal exchanged at maturity are usually different from those exchanged at the start)

there is no exchange of interest amounts but the differential between the spot and forward rates reflects differences in interest rates (in line with covered interest rate parity)



c) currency swap

currency swap is a contract which commits two counterparties to exchange over an agreed period two streams of interest payments in different currencies and usually also at the end of the contract to exchange the corresponding principal amounts at an exchange rate agreed at the start of the contract



terminology

currency (coupon) swap involves an exchange of interest streams of which both are at a fixed rate of interest

cross-currency swap involves an exchange of interest streams of which at least one is at a floating rate of interest

cross-currency coupon swap is a fixed-against-floating swap

cross-currency basis swap is a floating-against-floating swap

principal amounts can be exchanged at the start of the swap as well (particularly where the swap is associated with a new borrowing) but re-exchange would be at the original exchange rate (the amounts of principal exchanged at maturity will be the same as those exchanged at the start)

the agreed exchange rate may be the spot exchange rate prevailing at the start of the swap but not necessarily; it is subject to negotiations between the counterparties

currency swap is not a derivative instrument in a strict sense because there is an eventual movement of principal (a derivative is an instrument whose performance is derived from an underlying asset but does not require that asset to be bought or sold)

it is possible to construct a currency swap with no exchange of principal even at maturity (in that case a net settlement involves the compensation for paying the currency which appreciates over the life of the swap)

d) precursors of currency swaps

parallel loans involved a simultaneous provision of two loans denominated in two different currencies

the loans were treated as two separate transactions (there was no right to offset the obligations if one of the counterparties defaulted)

the transactions impacted on the balance sheet like conventional loans

back-to-back loans involved a simultaneous provision of two loans denominated in two different currencies with the right to offset but with still two separate sets of rights and obligations (this legal uncertainty increased by cross-border structure)

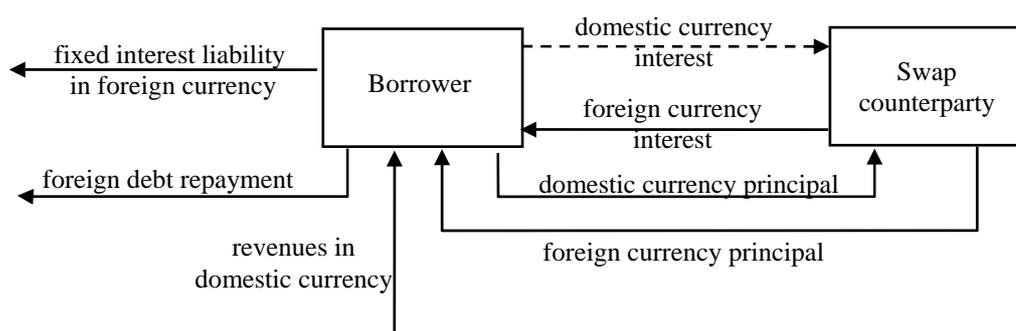
the above instruments originated as a means of circumventing exchange controls in the UK

9.2 Risk management with currency swaps

currency swaps create an exposure both to changes in exchange rates and interest rates so they can be used for hedging and taking the exchange rate risk, the interest rate risk or both

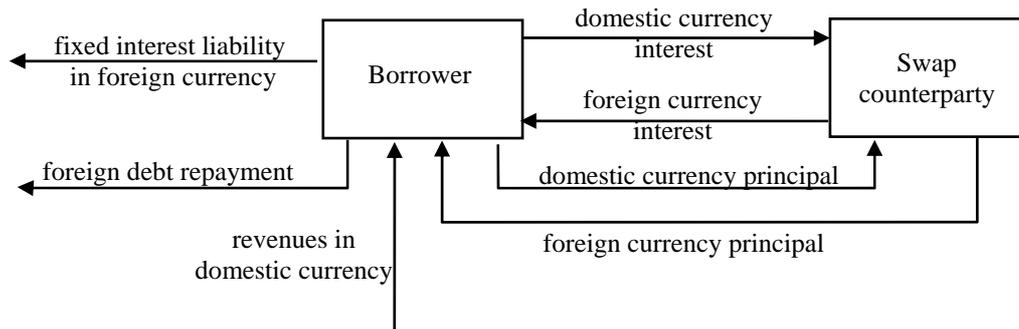
in practice currency swaps are used exclusively only to hedge and not to take currency risk (low liquidity makes it difficult to open and close speculative positions) and to hedge and take the exchange rate risk

i) borrowing in foreign currency



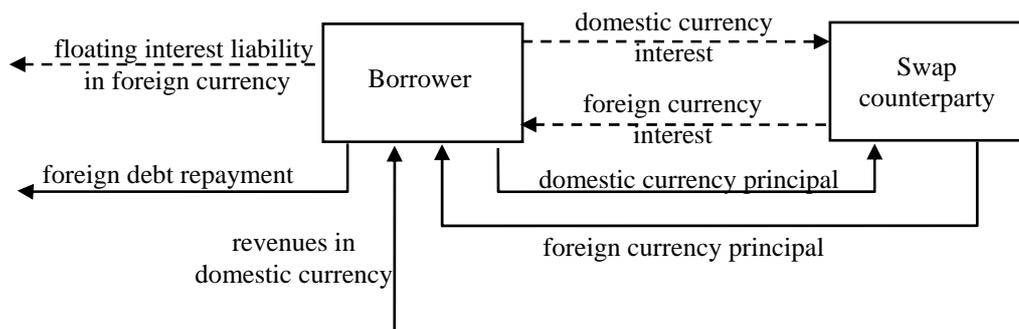
cross-currency coupon swap hedges fixed interest rate borrowing in a foreign currency which is expected to appreciate and at the same time enables to benefit from falling domestic currency interest rate

the borrower would pay floating interest in domestic currency from earnings generated by its business and in exchange will receive fixed interest in foreign currency which would be used to service the fixed interest foreign debt



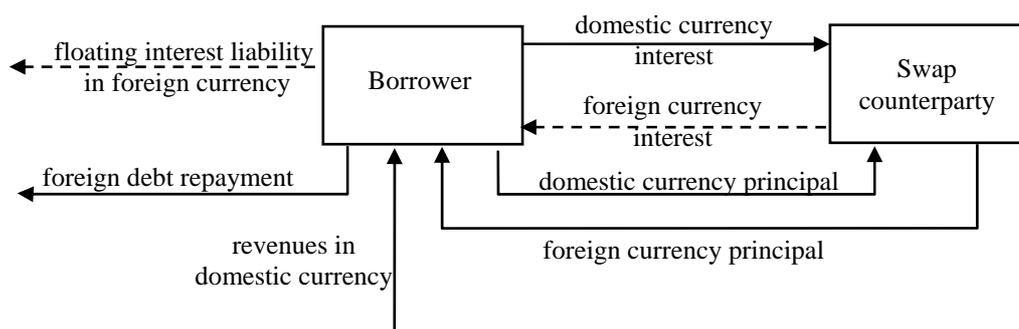
currency coupon swap hedges fixed interest rate borrowing in a foreign currency which is expected to appreciate and at the same time benefits from avoiding rising domestic currency interest rate

the borrower would pay fixed interest in domestic currency from earnings generated by its business and in exchange will receive fixed interest in foreign currency which would be used to service the fixed interest foreign debt



cross-currency basis swap hedges floating interest rate borrowing in a foreign currency which is expected to appreciate and at the same time enables to benefit from falling domestic currency interest rate

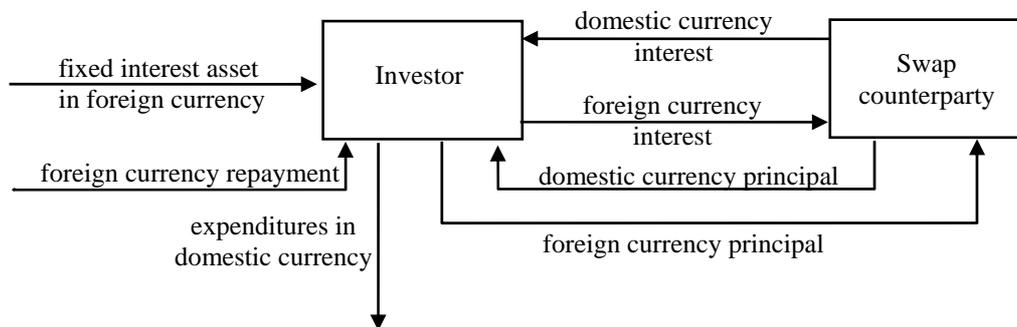
the borrower would pay floating interest in domestic currency from earnings generated by its business and in exchange will receive floating interest in foreign currency which would be used to service the floating interest foreign debt



cross-currency coupon swap hedges floating interest rate borrowing in a foreign currency which is expected to appreciate and at the same time benefits from avoiding rising domestic currency interest rate

the borrower would pay fixed interest in domestic currency from earnings generated by its business and in exchange will receive floating interest in foreign currency which would be used to service the floating interest foreign debt

ii) investing in foreign currency



currency coupon swap hedges fixed interest investment in a foreign currency which is expected to depreciate and at the same time benefits from avoiding falling domestic currency interest rate

the investor would receive fixed interest in domestic currency through the swap and in exchange pay fixed interest in foreign currency which it would fund from the interest received on its foreign borrowing

similar diagrams can be drawn for currency swaps which

- hedge a floating interest asset in a depreciating currency
- avoid a falling interest rate in domestic currency or
- benefit from a rising interest rate in domestic currency

9.3 New issue arbitrage

arbitrage opportunities may arise when differences exist between the interest paid (or received) through the swap and the interest received from (or paid by) another financial instrument

an arbitrage involving a currency swap generates cash flows in two different currencies that must be converted into the common denominator

the conversion is done in the forward foreign exchange market (to avoid currency risk)

equation of uncovered interest rate parity

$$\frac{S - F}{S} = \frac{r_B - r_V}{1 + r_V} \Rightarrow r_V = \text{BCF} \times r_B$$

r_B ... base currency's interest rate

r_V ... variable currency's interest rate

BCF ... basis conversion factors (regularly published and updated)



Take two companies which can issue five-year bonds denominated in EUR or USD at relative cost shown in the table:

	EUR	USD
Company A	6.5 %	4.0 %
Company B	7.0 %	5.2 %
differential	50 bp	120 bp
Arbitrage potential = 120 USD – 50 EUR		

company A can fund itself more cheaply in both markets \Rightarrow A has an *absolute advantage* in both markets and B has absolute disadvantage in both markets

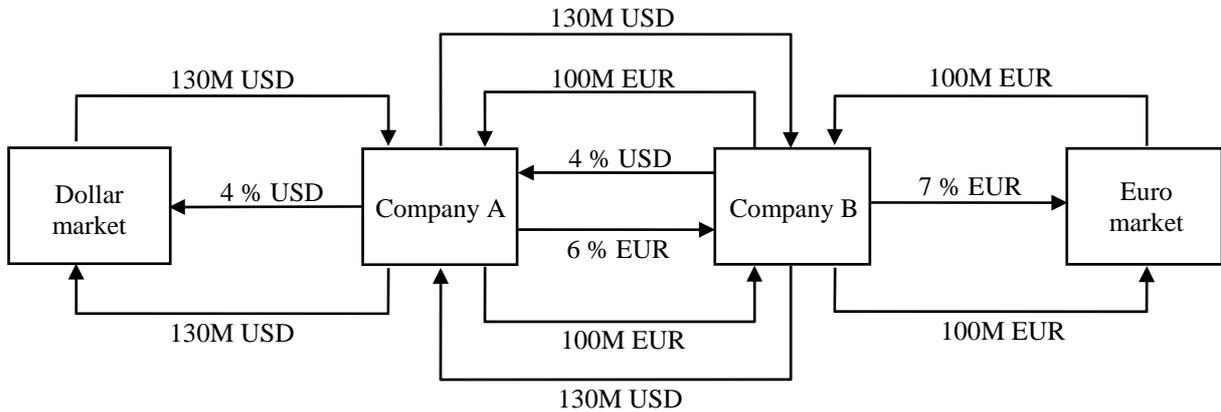
company A's interest rate advantage is greater in the USD market \Rightarrow A has a *relative advantage* in the USD market and *relative disadvantage* in the EUR market

company B's interest rate disadvantage is less in the EUR market \Rightarrow B has a *relative advantage* in the EUR market and relative disadvantage in the USD market

both firms can gain from putting on a currency swap if they want to end up with currency borrowing in the markets where they have comparative disadvantage (A needs to borrow euros while B needs to borrow dollars)

arrangement of a swap: both firms borrow funds in the markets where they have comparative advantage and then swap interest payments in such a way that both save borrowing cost relative to those achieved without the swap

at a current exchange rate USD/EUR = 1.3 the company A issues dollar denominated bonds in a principal amount of 130 million USD and the company B issues euro denominated bonds in a principal amount of 100 million EUR
assume that the conversion factor from euro to dollar is currently 0.9



net cost of funds to the company A:

paid interest on bonds	- 4 % USD
received interest through swap	+ 4 % USD
paid interest through swap	- 6 % EUR
net cost	- 6 % EUR

company A reduced the cost of its EUR funding by 50 basis points (= 7.0 – 6.5 %)

net cost of funds to the company B:

paid interest on bonds	- 7 % EUR
received interest through swap	+ 6 % EUR
paid interest through swap	- 4 % USD
net cost	- 4 % USD – 1 % EUR

company B pays 4 % USD interest plus 1 % EUR

because one euro basis point is equivalent to 0.9 dollar basis points the B's net cost of borrowing is $4 + 0.9 = 4.9 \% \text{ USD}$

company B reduced the cost of its USD funding by 30 basis points (= 5.2 – 4.9 %)

the two companies saved together

in EUR basis points = $50 + 30/0.9 = 83.3$

in USD basis points = $50 \times 0.9 + 30 = 75$

the two companies exploited the arbitrage potential

in EUR basis points = $120/0.9 - 50 = 83.3$

in USD basis points = $120 - 50 \times 0.9 = 75$



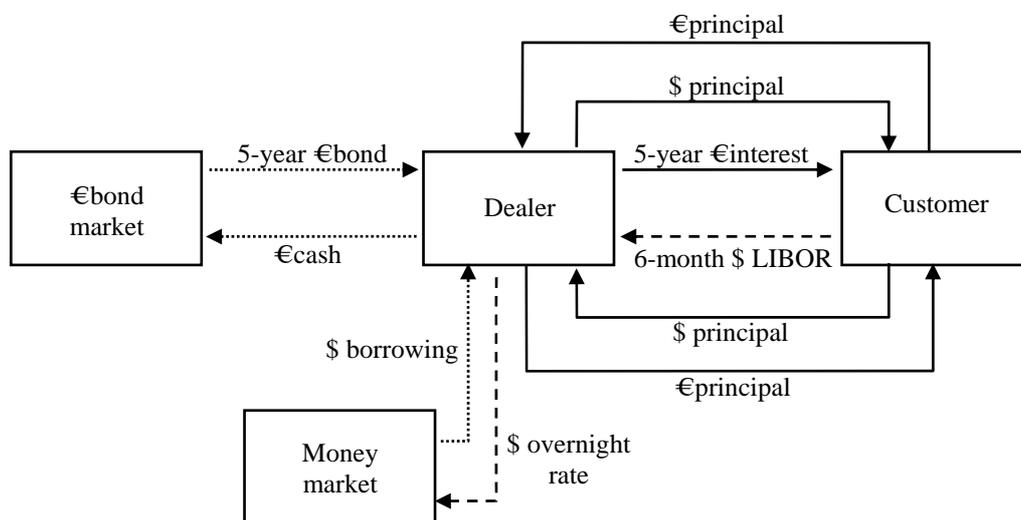
9.4 Warehousing currency swaps

warehousing are called hedging techniques that aim to absorb risk exposures from swaps

that arise before a matching currency swap is agreed

unhedged currency swaps involve both the interest rate risk and the currency risk

warehousing cross-currency coupon swap with initial exchange of principal amounts



risk exposures before a matching swap can be agreed

- a fall in euro interest rates would mean that the matching swap would pay less in fixed interest that is being paid out on existing swap
- a rise in dollar LIBOR would mean that until the next LIBOR refixing date more interest would be paid out on the floating leg of the matching swap than is being received on the existing swap
- a depreciation of the dollar against the euro would mean that a smaller principal amount of euros would be received through a matching swap at its maturity than would have to be paid out on the existing swap for the same amount of dollars

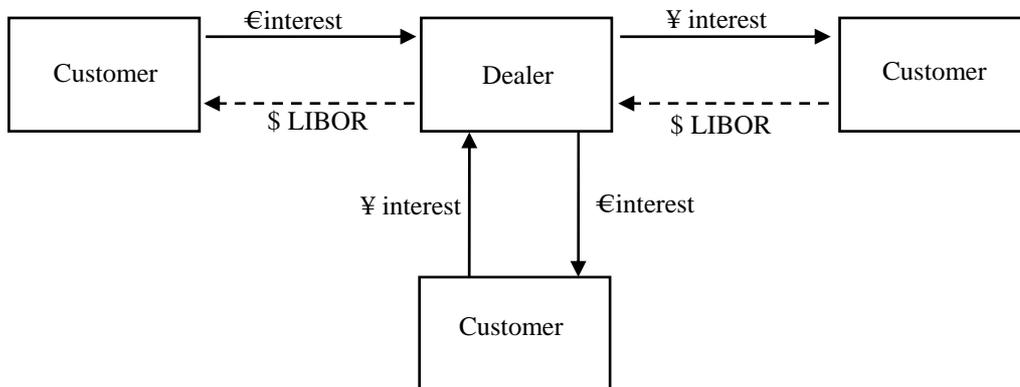
warehousing strategy

- buying five-year euro government bonds (using funds that were received in the initial exchange through swap)

- ⇒ any subsequent fall in euro interest rates will increase the price of the bonds (capital gain should be more or less equal to the income loss on the matching swap)
- ⇒ any subsequent depreciation of the dollar will increase the dollar value of the euro denominated bonds (the gain should offset the mismatch between amounts of currencies received and paid at maturity of the original and matching swaps)
- borrowing dollars to fund the payment of the principal amount in dollars in the initial exchange through the swap (using overnight money because of unknown warehousing period)
 - ⇒ the hedge involves the basis risk between the overnight rate paid for the borrowed dollars and the 6-month LIBOR received through the swap

hedging with a cocktail swap

cocktail swap is a combination of several different currency swaps, each of which offsets only part of the risk on the other swap, but which together form a fully hedged structure



9.5 Valuing currency swaps

value of a currency swap is the difference between net present values of the future cash flows to be paid and received through the swap where the value in one currency is translated into the other currency

$$V = \text{NPV}(\text{currency A}) - \frac{\text{NPV}(\text{currency B})}{\text{exchange rate (A/B)}}$$

net present value of each future cash flow is calculated by standard rules of discounting

the value of a generic swap priced at current market rates should be zero when the swap is originated; the swap tends to acquire a non-zero value after it is transacted as their exchange rates and interest rates become outdated



A currency swap has three years remaining to maturity. It is a swap with the final exchange of principal amounts of 150 mil USD and 100 mil EUR. The dollar interest stream pays a fixed rate of 4 % and the euro interest stream pays a fixed rate of 10 %. The current three-year dollar swap rate is 5 % and the current three-year euro swap rate is 9 %. Determine the actual value of the swap under assumption that the actual USD/EUR exchange rate is 1.45.

current value of the dollar stream =

$$(0.04 \times 150\,000\,000) \times \sum_{t=1}^3 \frac{1}{1.05^t} + \frac{150\,000\,000}{1.05^3} = 145\,915\,128 \text{ USD}$$

current value of the euro stream =

$$(0.1 \times 100\,000\,000) \sum_{t=1}^3 \frac{1}{1.09^t} + \frac{100\,000\,000}{1.09^3} = 102\,531\,295 \text{ EUR} = 148\,670\,378 \text{ USD}$$

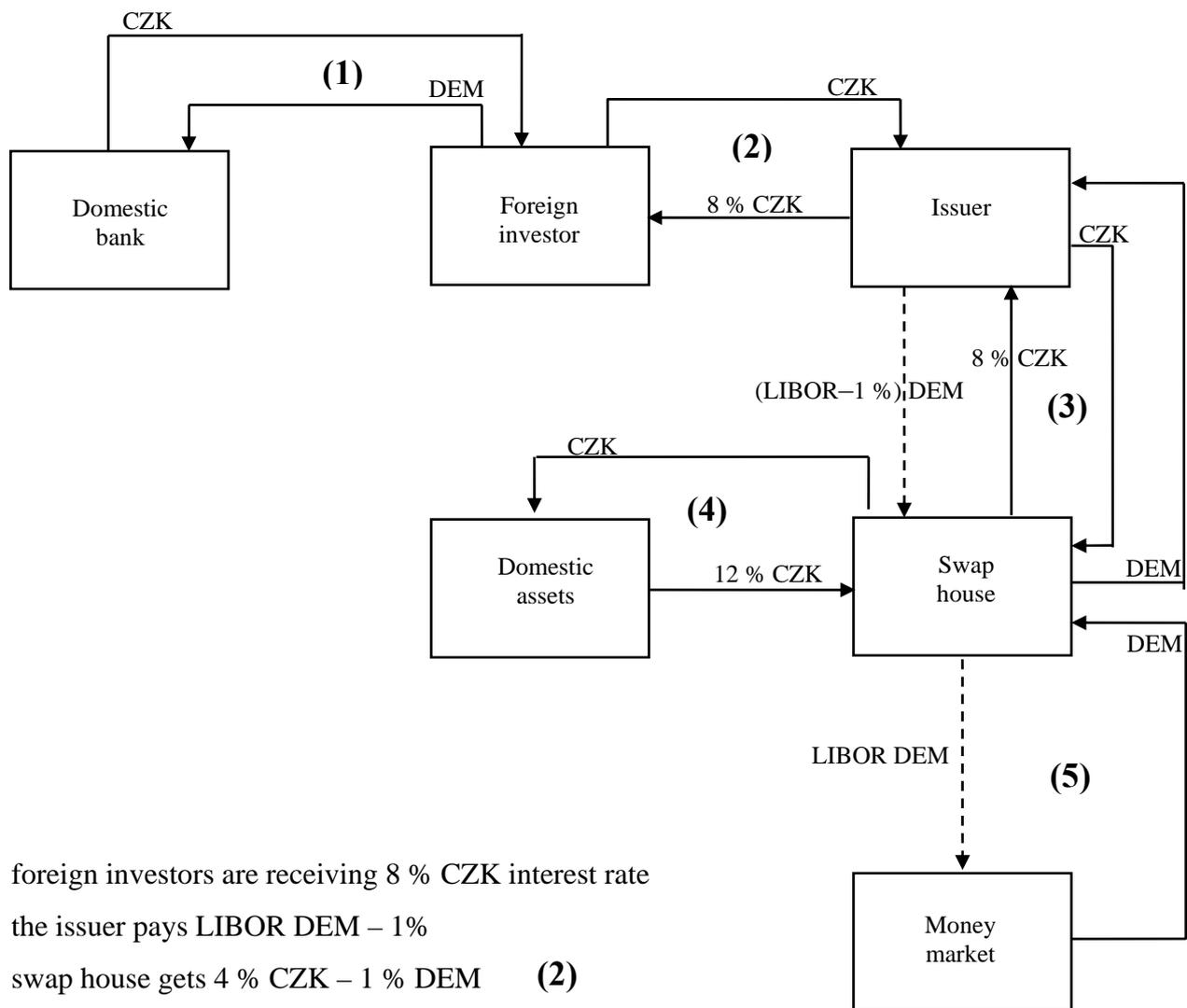
value of the swap = $148\,670\,378 - 145\,915\,128 = 2\,755\,250$ USD (for the counterparty which receives the euro stream and pays the dollar stream)



9.6 Two case studies

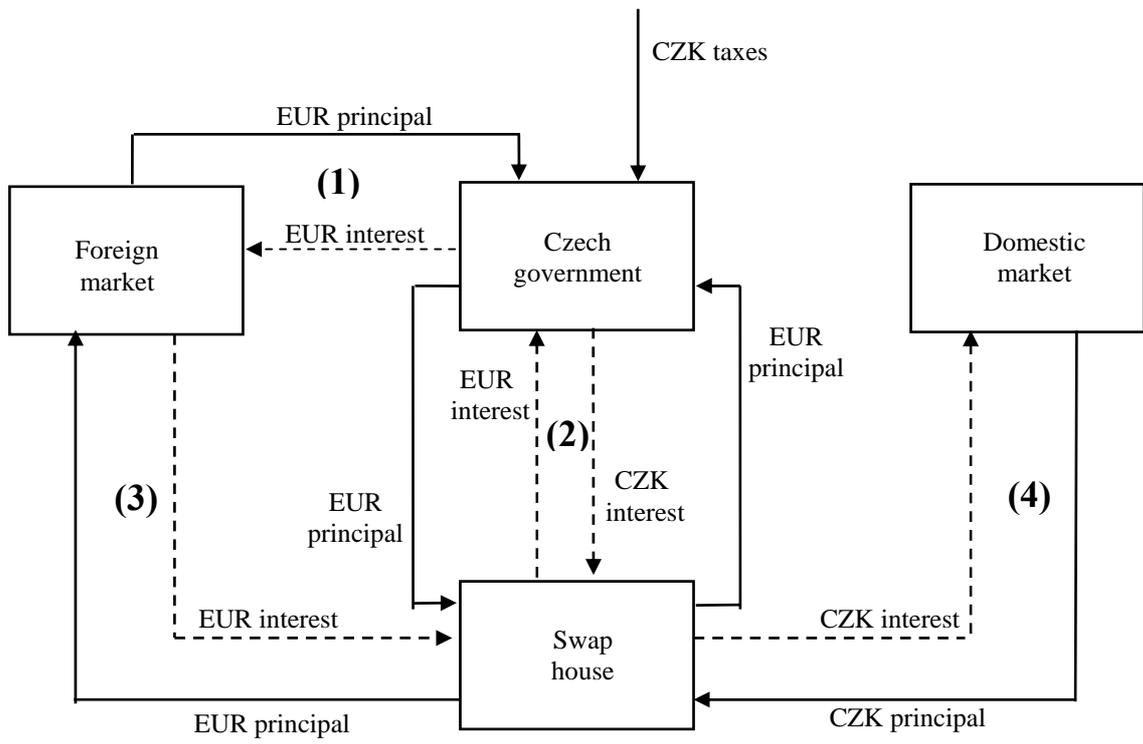
a) the surge of CZK euro-bonds in the first half of 1990s

1. Foreign investors buy CZK (sell DEM), CZK/DEM exchange rate appreciates
2. Foreign investors buy CZK-eurobonds from the issuer in exchange for CZK return
3. The issuer arranges a swap deal (coupon cross-currency swap) with a swap house with the aim to have a net liability in DEM
4. The swap house buys domestic assets (Treasury bills, extension of credit, bank deposit)
5. The swap house raises DEM funds in the foreign exchange market



b) external borrowing without an appreciation pressure on domestic currency

1. The Czech government borrows in EUR (issues foreign bonds denominated in euro) and pays the EUR interest
2. The swap house arranges a cross-currency swap with the Czech government (the Czech government ends up paying CZK interest)
3. The swap house invests EUR in the foreign exchange market (there is no conversion of EUR into CZK, thereby no impact on the CZK/EUR exchange rate)
4. The swap house raises CZK funds in the domestic market (a greater demand for CZK borrowing may push CZK interest rates up)

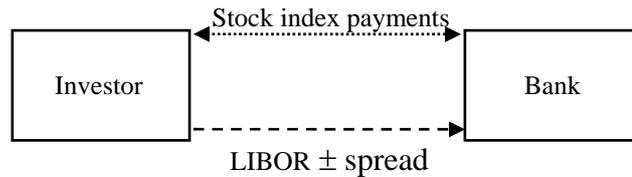


X. EQUITY SWAPS

equity swap is a contract which commits two counterparties to exchange, over an agreed period and with a given frequency, two streams of payments:

- i) payments linked to percentage changes in an agreed stock market index
- ii) payments linked to an agreed short-term interest rate

both income streams are applied to an agreed notional principal amount



payments in the stock index stream can go either way (if the stock index decreases the investor must pay the bank an amount equivalent to the decrease in the index and vice versa)

there is no exchange of principal (this is used only in the calculations of payments to be exchanged)

it is market practice to net individual payments so that only one payment is actually made between counterparties

equity swap is classed as a derivative instrument (it makes payments derived from a cash instrument) and as an off-balance sheet instrument (it does not impact on the balance sheet)

equity swaps are over-the-counter products whose specifications are negotiated by the swap counterparties

10.1 Examples of equity swaps

a) equity swap with a fixed notional principal amount



An equity swap has been agreed with the following specifications:

currency	sterling
notional amount	100 million £
stock market index	FT-SE 100
interest rate	six-month £ LIBOR
spread to investor	10 basis points

payment frequency semi-annual
maturity two years

The following table summarizes the cash flows through the equity swap.

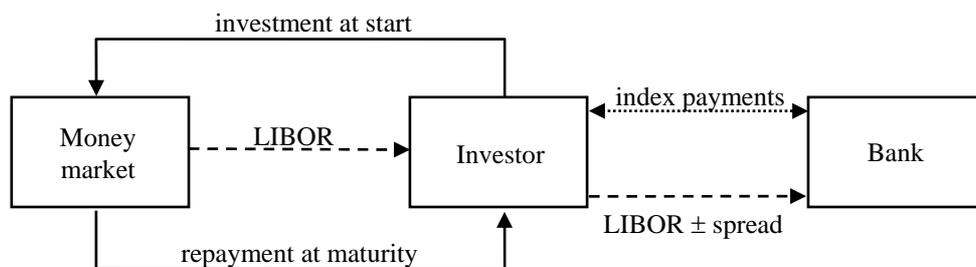
Month	Number of days	FT-SE	FT-SE change	LIBOR -10 p.b.	Index payment	Interest payment
0		2149,40		13,00		
6	182	2499,50	16,29	11,94	16290000	5953644
12	183	2420,20	-3,17	10,63	-3170000	5329562
18	183	2707,60	11,88	10,06	11880000	5043781
24	183	2778,80	2,63	10,00	2630000	5013699
Total					27630000	21340685

The investor has made a total profit by the end of the swap of:

$$\text{profit} = 27\,630\,000 - 21\,340\,685 = 6\,289\,315 \text{ £}$$



equity swap can be used as a special asset swap which transforms a money market investment into a synthetic equity investment (it simulates investment in a stock market without the need to invest principal in this market)



$$\text{net income} = \text{index payment} \pm \text{spread}$$

advantages:

- a diversified portfolio of shares is purchased in a single transaction that allows avoiding transaction costs associated with direct trading in the stock market
- index-tracking investment strategy is simplified by avoiding the need to rebalance the managed portfolio in conformity with changes in the benchmark portfolio
- investment restrictions can be circumvented

b) equity swap with a variable notional principal amount

the notional principal amount varies with changes in the agreed stock market index
the principal amount is either increased by the amount equal to the index payment received
or reduced by the amount equal to the index payment paid



Make up the cash flow streams of an equity swap with specifications taken from the previous example assuming that the notional principal amount varies with changes in the stock index.

Month	FT-SE change	LIBOR -10 p.b.	Index payment	Interest payment	Principal amount
0		13,00			100000000
6	16,29	11,94	16290000	5953644	116290000
12	-3,17	10,63	-3686393	6197747	112603607
18	11,88	10,06	13377309	5679479	125980916
24	2,63	10,00	3313298	6316303	129294214
Total			29294214	24147174	

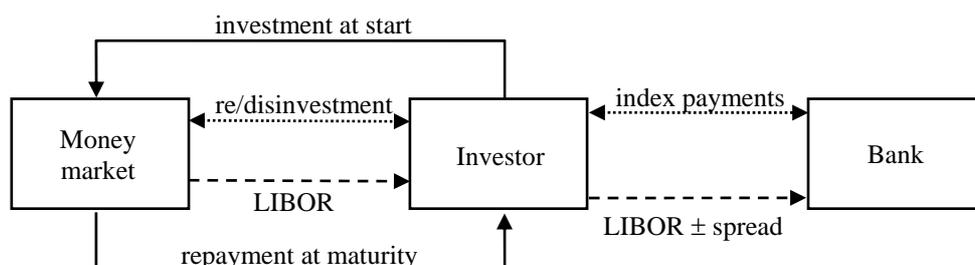
The investor has made a total profit by the end of the swap of:

$$\text{profit} = 29\,294\,214 - 24\,147\,174 = 5\,147\,040 \text{ £}$$



equity swap with variable notional principal amount can be used to simulate a stock market investment in which profits and losses are capitalised (profits are reinvested and losses are met through disinvestment)

equity-linked payments received are used to buy additional cash assets and equity-linked losses paid are funded by drawing down existing cash assets

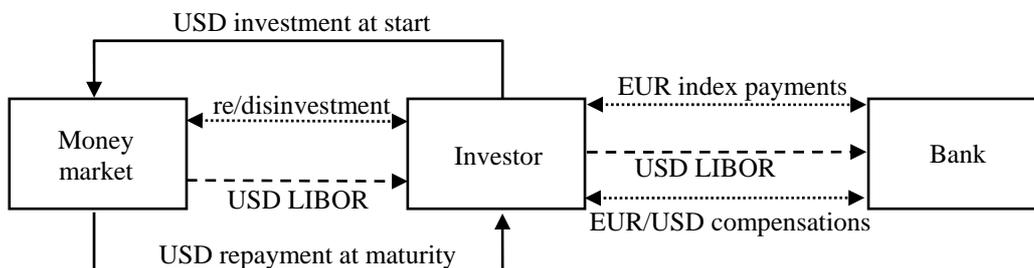


c) cross-currency equity swap

in a cross-currency equity swap the cash flow streams are denominated in two different currencies

if the purpose of the swap is to simulate an investment into a foreign stock index there is usually an additional stream of payments that takes account of exchange rate changes (they can go thus in either direction)

the aim is to match the change in the value of the actual principal amount of the underlying investment with the notional principal amount in the equity swap



if the euro appreciates against the dollar then the dollar value of the notional principal amount of the swap (which is denominated in EUR) becomes larger than the actual principal amount of the dollar investment (the investor would receive less dollars than he would pay through the swap)

the bank is required to compensate the investor by making a payment equal to the difference between the two dollar payments

if the euro depreciates against the dollar then the dollar value of the notional principal amount of the swap (which is denominated in EUR) becomes smaller than the actual principal amount of the dollar investment (the investor would receive more dollars than he would pay through the swap)

the investor is required to compensate the bank by making a payment equal to the difference between the two dollar payments

10.2 Esoteric swaps

esoteric swaps (non-generic, exotic swaps) are swaps which differ in one or more characteristics that define generic (plain vanilla, straight) swaps

generic characteristics: constant principal amount, constant fixed interest rate, regular payments of interest, immediate start of the deal, no special risk features

a) swaps with variable principal amounts

amortising swaps have principal amounts which decrease in steps over the life of the swap

(they are used in conjunction with instruments with amortising redemption structures)

accreting swaps have principal amounts which increase in steps over the life of the swap

(they are used in conjunction with some form of project financing facilities)

roller-coaster swaps are a combination of an amortising and accreting swap

b) swaps with variable fixed interest

step-up/step-down swaps are swaps whose fixed interest rate increases or decreases in predetermined steps

spread-lock swaps (deferred rate-setting swaps) are swaps in which the fixing of the fixed rate (usually in terms of a benchmark yield and a spread) is deferred until later during the life of the swap

c) swaps with irregular interest payments

deferred-coupon bond swaps have cash flow designed to match fixed-income bonds which pay no coupon for the first years of the bond, coupons are then paid normally over the remaining years of the bond

zero-coupon swaps are swaps in which there is only one payment of interest made by the counterparty which would be the payer of fixed interest (they are used in conjunction with underlying zero-coupon securities)

premium/discount swaps involve fixed interest cash flows at off-market interest rates (above market levels in case of premium swaps and below market levels in case of discount swaps); the differential with current swap rates is offset by an upfront payment

d) swaps with deferred start dates

delayed-start swaps allow for agreeing the terms of the transaction now but the deal will not come into effect until a future date

forward swaps are delayed-start swaps which come into effect by more than six months after the deal is agreed (as much as three years)

10.3 Swaption

swaption is an option to enter into a forward coupon swap

call swaption is the right to buy a forward swap and thus to pay fixed interest
 put swaption is the right to sell a forward swap and thus to receive fixed interest



a company with a five-year bank loan pays a six-month interest priced at LIBOR plus 100 basis points and is concerned that short-term interest rates are likely to be very volatile in one year time

a company can fix its borrowing rate for the remaining four-year period of the loan by putting on a call swaption with an exercise rate of 11 % for which it pays a premium of 1 %

effective borrowing rate if the swaption is exercised

fixed interest through swap	- 11 %
floating interest through swap	+ LIBOR
borrowing rate on bank loan	- (LIBOR + 100 bp)
swap premium	- 100 bp

the company decided to absorb any increase in its borrowing cost up to 13 %



XI. CREDIT DERIVATIVES

11.1 Basic notions

credit derivatives are financial instruments that are designed to transfer the credit exposure on an underlying asset between two or more parties

benefits of credit derivatives

- the ability to isolate credit risk and manage it independently of the exposure to an underlying asset
- the efficient exchange of credit risk because of savings in transaction cost (the alternative is buying or selling referenced assets)
- preservation of undisturbed business relationship with clients (liquidation of assets may sour the customer relationship)

parties of the contract

credit protection buyer = credit risk seller

credit protection seller = credit risk buyer

types of credit risk:

default risk means that the debtor (issuer of a bond, receiver of a loan) will not repay the outstanding debt in full

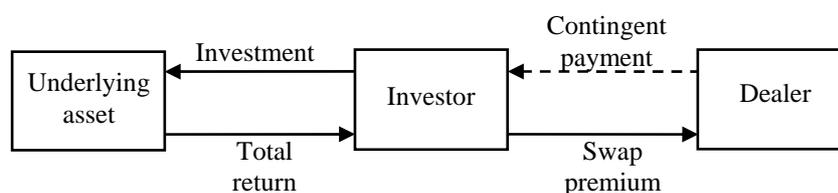
downgrade risk means that a recognized rating agency will lower its credit rating for a debtor based on an evaluation of the debtor's current earning power to pay the debtor's fixed income obligations

credit spread risk means that the market's reaction to perceived credit deterioration will manifest itself in an increased spread over a reference rate

11.2 Credit swaps

credit swaps are credit derivatives that are designed in form of a swap contract

a) credit default swap (CDS)



from the investor's perspective the primary purpose of the CDS is to hedge the credit exposure to a referenced asset or an issuer

the dealer has effectively insured the investor against credit losses on the underlying asset

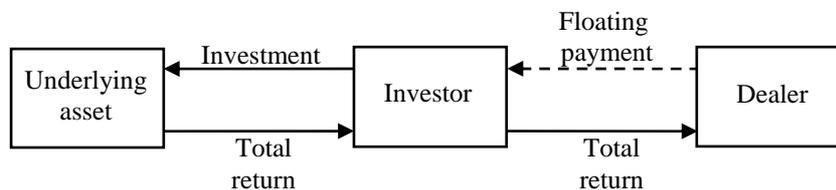
structure of cash flows:

- investor (credit protection buyer) pays a fee (swap premium) to the dealer (credit protection seller) in return for the right to receive a payment conditional upon the default on an underlying asset
- the investor who is the owner of the underlying asset continues to receive the total return on the underlying asset (whether it is positive or negative)
- if no credit event has occurred (missed coupon, downgrade, merger, etc.) by the maturity of the swap then both sides terminate the agreement and no further obligations are incurred
- the fee may be a single bullet payment or a floating rate benchmarked to LIBOR
- the contingent payment may be a predetermined fixed amount or may be determined by the decline in value of the underlying asset

b) total return credit swap (TROR)

from the investor's perspective the TROR may be used both to hedge the credit exposure as well as to increase credit exposure to a referenced asset or an issuer

i) hedging credit exposure

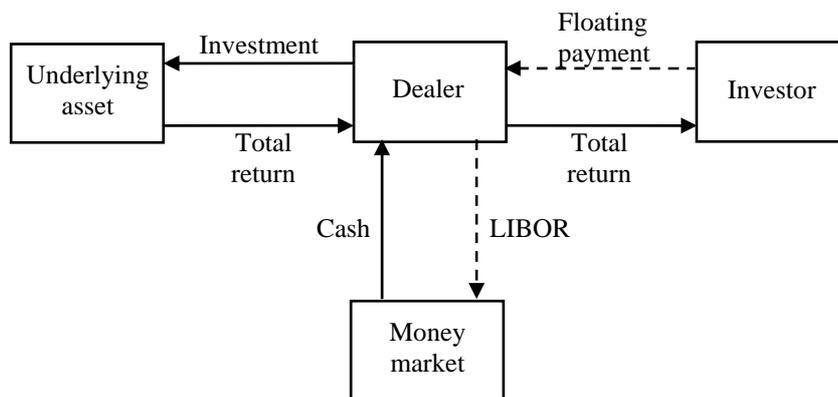


structure of cash flows:

- the investor keeps the underlying asset on his balance sheet
- investor (credit protection buyer) pays the total return of the underlying asset (whether it is positive or negative) to the dealer (credit protection seller) in return for receiving regular floating payments

- in case of negative return the investor receives a payment equal to the amount of the negative return in addition to the floating payment (he is reimbursed for the loss in asset value)

ii) *increasing credit exposure*



the investor is the receiver of total return on the underlying asset without the need to own this asset

the TROT exposes the investor both to credit risk (changing credit spreads) and market risk (changing market rates); a credit gain may be offset by a market loss

structure of cash flows:

- the investor (credit risk buyer) receives the total return on the underlying asset including the capital appreciation or depreciation
- in case of negative total return the investor pays any depreciation of the underlying asset to the credit risk seller (alternatively the investor may transfer collateral as security to the dealer from which any depreciation of the underlying asset is covered)
- the dealer may not initially own the underlying asset before the swap is transacted, he usually borrows capital to purchase the asset (borrowing cost is factored to the floating rate)
- from the dealer's perspective all of the cash flows net out to a fixed spread over LIBOR (his position is neutral)

benefits of credit swaps over holding the underlying asset

a) *lower transaction costs*

the same economic exposure can be achieved in one swap transaction as opposed to several cash market transactions (arranging a loan, purchase and sale, etc.)

b) *customization*

credit swaps are privately negotiated contracts that can be tailored to meet investors' particular needs

a rich choice of underlying assets (junk bonds, bank loans or any other fixed income securities)

the tenor of the swap can be set for a shorter time horizon than that of the underlying asset



A company has issued a high yield bond at par with a 5-year maturity and a credit spread of 300 basis points over the 5-year Treasury (currently 6.25 %). The bond thus pays an annual coupon of 9.5 % which is equal to the bond's discount rate. An investor believes that this spread will decline over the next year. He concludes a 1-year TROR in which he receives the total return on the bond (coupon plus any price appreciation or depreciation). In return he pays a rate equal to the 1-year Treasury (currently 5.7 %) plus 30 basis points. The notional amount of the swap is 20 million USD.

Assume that after one year the credit spread on the company's bond decreased to 275 basis points and the Treasury yield curve remained unchanged (the 4-year Treasury is 6.2 %).

Cash flow from the swap:

$$\text{coupon payment} = + 20\,000\,000 \times (0.0625 + 0.03) = 1\,850\,000$$

$$\text{swap premium} = - 20\,000\,000 \times (0.057 + 0.003) = 1\,200\,000$$

price appreciation =

$$20\,000\,000 \times \left(\sum_{t=1}^4 \frac{0.095}{(1 + 0.062 + 0.0275)^t} + \frac{1}{(1 + 0.062 + 0.0275)^4} - 1 \right) = +356\,761$$

$$\text{total gain} = 1\,850\,000 - 1\,200\,000 + 356\,761 = 1\,006\,761 \text{ USD}$$

Assume alternatively that the investor guessed right on the declining credit spread but the market experienced a general rise in interest rates (the 4-year Treasury is now 7.25 %).

price depreciation =

$$20000000 \times \left(\sum_{t=1}^4 \frac{0.095}{(1 + 0.0725 + 0.0275)^t} + \frac{1}{(1 + 0.0725 + 0.0275)^4} - 1 \right) = -316987$$

$$\text{total gain} = 1850000 - 1200000 - 316987 = 333013 \text{ USD}$$



c) *leverage*

credit swap requires a much smaller capital commitment than the outright purchase of an underlying asset to access the returns on the underlying asset

a degree of leverage is measured by a leverage factor (e.g. 10 : 1)

leverage can significantly boost the return to invested capital but if the underlying asset declines in value the losses can pile up quickly



Compare returns on the two investment alternatives: a) purchasing and holding a bond or b) arranging a total return swap on the underlying bond.

a) The investor purchases a bond priced at par with a coupon of 6 % in a nominal value of 10 mil USD. The bond is held for three years. Suppose that the yield curve remains unchanged so the bond can be sold at par. The return on the investment is 6 %. The result can be verified by solving the equation

$$10000000 = \frac{600000}{1+R} + \frac{600000}{(1+R)^2} + \frac{10600000}{(1+R)^3} \Rightarrow R = 0.06$$

Suppose alternatively that the yield curve shifts upwards, so the bond portfolio suffers a capital loss of 1 million USD. This scenario gives a lower but still positive return of 2.76 %.

The result can be verified by solving the equation

$$10000000 = \frac{600000}{1+R} + \frac{600000}{(1+R)^2} + \frac{9600000}{(1+R)^3} \Rightarrow R = 0.0276$$

b) The investor arranges a three-year TROR in a notion amount of 10 mil USD. He is receiving the bond's coupon of 6 % in exchange for paying 315 basis points above a three-year Treasury. The investor is required to put up collateral in terms of 1 mil three-year Treasury (the leverage factor is 10). The Treasury's interest belongs to the investor. The investor is thus receiving an interest rate of

$$6 \% + \text{Treasury} - (\text{Treasury} + 3.15 \%) = 2.85 \%$$

When the yield curve remains unchanged the investor is given back the full value of collateral. The rate of return on the TROR is 28.5 %. The result can be verified by solving the equation

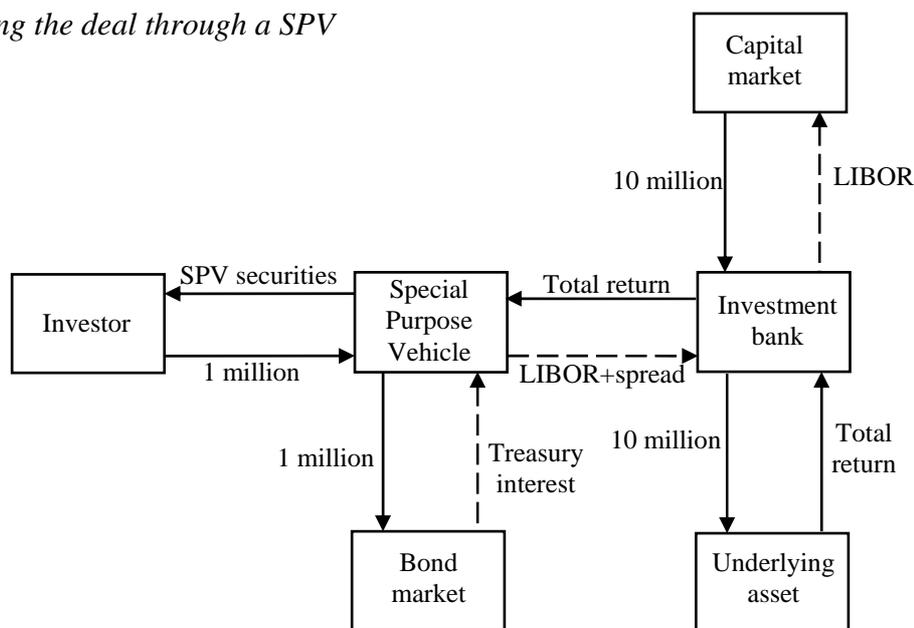
$$1000000 = \frac{285000}{1+R} + \frac{285000}{(1+R)^2} + \frac{1285000}{(1+R)^3} \Rightarrow R = 0.285$$

When the yield curve shift incurs a 1 million price depreciation of the underlying bond portfolio, the investor loses all collateral (he pays the negative return on the bond). The TROR yields a negative return of 7.4 %. The result can be verified by solving the equation

$$1000000 = \frac{285000}{1+R} + \frac{285000}{(1+R)^2} + \frac{285000}{(1+R)^3} \Rightarrow R = -0.074$$



Structuring the deal through a SPV



Investor

- investor purchases SPV securities at a nominal value of 1 million USD
- return on SPV securities = total return on the pool of underlying assets (based on a notional amount of 10 million USD) plus interest on Treasury notes (based on a nominal amount of 1 million USD) minus (LIBOR + spread + fee)
- investor does not enter into any derivative contract (conventional purchase of securities, no off-balance sheet operation)
- the contract is highly leveraged (investor committed only 1 million of capital but receives income on 10 million pool)

Special Purpose Vehicle

- SPV is a special purpose corporation set up by investment bank with the aim to issue securities to investors whose returns are tied to an underlying pool of assets
- the proceeds from the sale of the SPV securities are used to purchase Treasury notes
- the Treasuries serve as collateral which pays for the decline in value of referenced pool of assets
- SPV earns a fee, all other cash flows cancel out

Investment bank

- the bank borrows 10 million USD in capital market and uses this amount to purchase the pool of underlying assets
- total return on the pool is passed on the SPV under the swap agreement
- the bank earns a spread over LIBOR, all other cash flows cancel out

11.3 Credit options

credit options give the investor the right (but not the obligation) to buy (a call option) or sell (a put option) the specific credit risk at the exercise price until expiration of the option

the only risk is losing the cost of the option (paid option premium)

credit options are constructed to protect against credit risk

two types:

- credit level option (written on an underlying asset)
- credit spread option (written on a spread over a referenced riskless asset)

a) credit level option

the pay-off is determined by subtracting the market price of the bond from the exercise price or vice versa

long credit level put is an option where the option writer agrees to compensate the option buyer for a decline in the value of a financial asset below a specified exercise price determined in terms of an acceptable spread over the reference asset

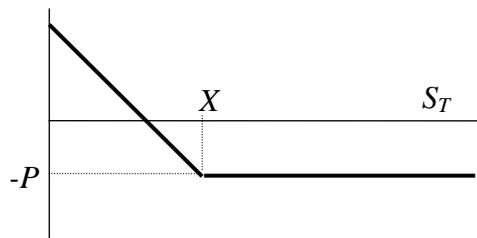
$$V = \max(0, X - S_T) \text{ where}$$

$X = \sum_t \frac{CF_t}{(1+r+s)^t}$ is an exercise price specified as the bond's price that

corresponds to a given credit spread over a referenced asset

S_T is the market value of a bond at the maturity of the option

Credit level put option



A portfolio manager purchased a 5-year zero-coupon BB bond in a nominal value of 1000\$ at a credit spread of 215 basis point to comparable Treasury which is currently traded at a yield of 6.25 %. To be protected against a widening of the credit spread the manager also purchases a 1-year American credit put option whose exercise price is established at a credit spread of 225 basis points.

The exercise price of the option is thus set equal to

$$X = \frac{1000}{(1 + 0.0625 + 0.0225)^4} = 722 \$$$

It the bond is traded below this value the manager can exercise the option and receive the difference between 722 \$ and the market value of the bond.



long credit level call would be an option where the option writer agrees to compensate the option buyer for a rise in the value of a financial asset above a specified exercise price determined in terms of an acceptable spread over the reference asset

b) credit spread option

the pay-off is determined by taking the difference in the credit spreads multiplied by a specified notional amount and by a risk factor

long credit spread call is an option where the option writer agrees to compensate the option buyer when the credit spread exceeds the specified spread level

$V = \max(0, (s_T - x) \times M \times rf)$ where

x ... the specified exercise spread over the riskless asset

s_T ... the spread over the riskless rate at the maturity of the option

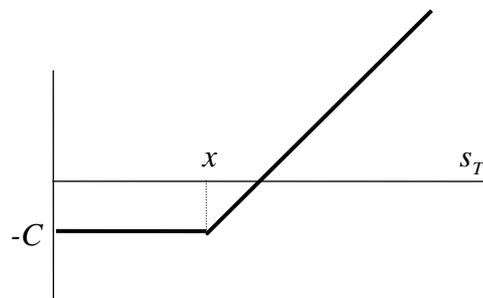
M ... notional amount of the swap

rf ... risk factor = duration + convexity (the construction of the pay-off is

based on the relationship

$$\Delta P = -\Delta r \times P \times (\text{duration} + \text{convexity})$$

Credit spread call option



A portfolio manager believes that the current credit spread for a given bond which is currently 178 basis points over the Treasury will increase to 250 basis points during the course of the next year. He buys a credit spread call with the tenor of one year, option premium of 125 basis points, risk factor of 6.65 and nominal value 20 million USD.

If the credit spread does indeed widen to 250 basis points at maturity of the option the manager will earn a profit of

$$20000000 \times [(0.025 - 0.0178) \times 6.65 - 0.0125] = 707600 \text{ USD}$$



11.4 Credit forwards

credit forwards have features of standard forward contracts that are adapted to handling credit risk

the purchaser of a credit forward receives the upside appreciation of the underlying asset but also shares its depreciation

in comparison with credit options credit forwards require no upfront payment but do not limit the potential loss

credit forwards may be constructed either on asset value or on credit spreads (like credit options)

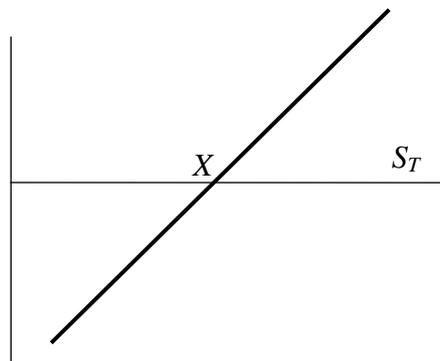
credit level forward

$$V = S_T - X$$

credit spread forward

$$V = (s_T - x) \times M \times rf$$

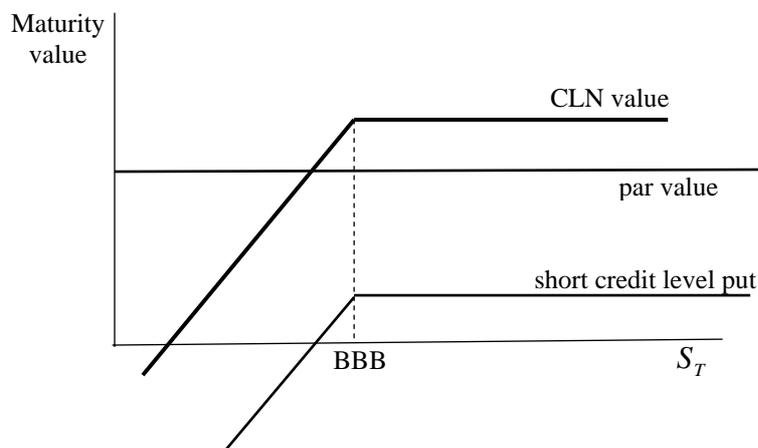
Credit level forward



11.5 Credit linked notes (CLN)

CLN are cash market instruments which combine the elements of a debt instrument and credit derivatives (they have embedded credit forwards and credit options)

a note with embedded short credit put option

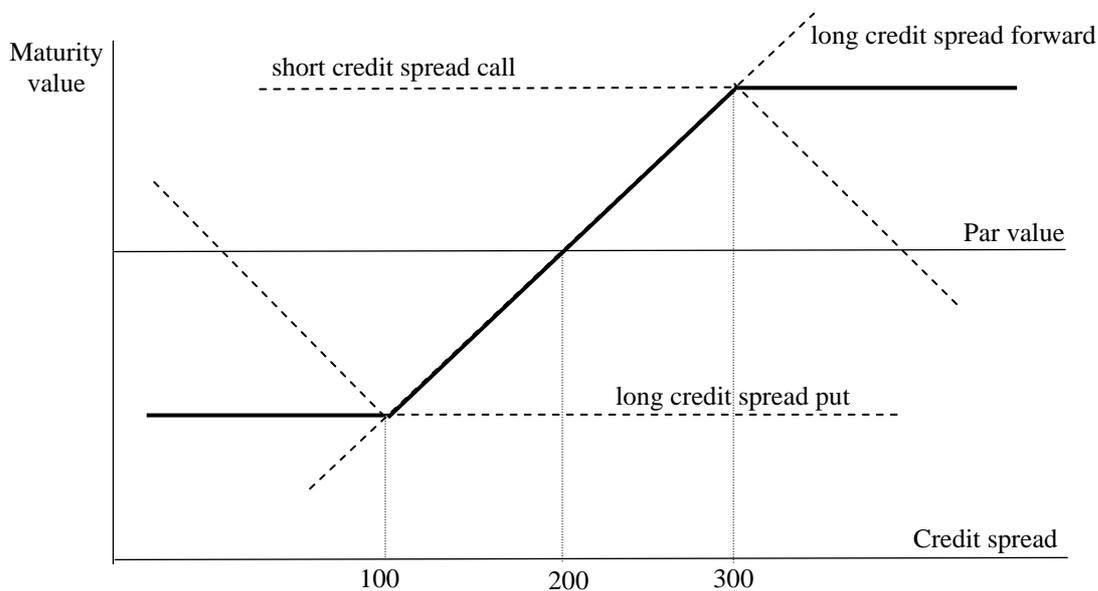


investor buys a note with embedded short credit level put option written on the credit rating BBB of the issuer (the notional amount of the option is the same as the face value of the note)

CLN provides a higher maturity value (the short position in the embedded option can be interpreted that the investor sold to the issuer of the note an option whose premium provides an incremental income)

if the credit rating on the referenced note declines the investor receives a lower maturity value (the embedded put was exercised)

a note with embedded credit forward and credit option



the current spread is 200 basis points and the note has been embedded with following credit derivatives:

- short credit spread call struck at 300 basis points
- long credit spread put struck at 100 basis points
- long credit spread forward priced at a current note spread of 200 basis points

if the spread widens the investor will receive at maturity a payment in excess of the par value; the appreciation of the note is capped at a spread of 300 b.p.

if the spread narrows the principal value of the note returned at maturity will decline; the depreciation of the note is stopped at a spread of 100 b.p.

between the spread range of 100 and 300 b.p. the principal value of the note is allowed to fluctuate