Funkce dvou proměnných

1. U funkcí $f(x,y)$ určete
   $\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y^2}, \quad \frac{\partial^2 f}{\partial x \partial y}$:

   $a)\ f(x,y) = x^3 + 2x^2y^2 - 3xy^3 + y^4 + 2$

   $b)\ f(x,y) = (x^2+2).\sin(x+3y)$

2. $a)$ Pro $f(x,y) = cos^2 2x.sin3y$ určete $\frac{\partial^2 f}{\partial x \partial y}$ v bodě $\left[\frac{\pi}{8}, 0\right]$

   $b)$ Pro $f(x,y) = arctg\frac{1+x}{1+y}$ určete $\frac{\partial^2 f}{\partial x^2}$ v bodě [0,1]

   $c)$ Pro $f(x,y) = \frac{x^2y+2y^2}{x-y}$ určete $\frac{\partial^2 f}{\partial y^2}$ v bodě [1,3]

   $d)$ Pro $f(x,y) = \sqrt{x+\frac{1}{y}}$ určete $\frac{\partial^2 f}{\partial x \partial y}$ v bodě $\left[5, \frac{1}{4}\right]$

3. Určete obecnou rovnici tečné roviny v bodě T:

   $a)\ f(x,y) = xy^2, \qquad T_0 = [-2,1] \in D_f$

   $b)\ f(x,y) = x^2 + y^2, \qquad T_0 = [0,0] \in D_f$

   $c)\ f(x,y) = x.e^{x^2y}\ \ [f(x,y) = x.\exp(x^2y)], \qquad T_0 = [-1,0] \in D_f$

   $d)\ f(x,y) = x^3 + 2x^2y + xy^2 - y + 1, \qquad T_0 = [1,0] \in D_f$

4. Vypočtěte $\overrightarrow{grad}\, f(x,y)$ v bodě:

   $a)\ f(x,y) = \frac{x}{y-x}, \qquad A = [1,2]$

   $b)\ f(x,y) = x^3y^2 - y + 1, \qquad B = [1,1]$

   $c)\ f(x,y) = \cos(\pi - x)\,.\,(y+1), \qquad C = \left[\frac{\pi}{2}, 0\right]$

5. Určete lokální extrémy funkce:

   $a)\ f(x,y) = x^2 + y^2 + 5$

   $b)\ f(x,y) = 5x^2 + 3xy + y^2 + 4x - y + 6$

   $c)\ f(x,y) = x^4 + y^4 + 2x^2y^2 - 4x$

Výsledky:

1. $a)$ $\dfrac{\partial^2 f}{\partial x^2} = 6x + 4y^2, \qquad \dfrac{\partial^2 f}{\partial y^2} = 4x^2 - 18xy + 12y^2,$

$$\frac{\partial^2 f}{\partial x \partial y} = 8xy - 9y^2$$

$b)$ $\dfrac{\partial^2 f}{\partial x^2} = 4x.\cos(x + 3y) - x^2 \sin(x + 3y)\,,$

$$\frac{\partial^2 f}{\partial y^2} = -9(x^2 + 2)\sin(x + 3y)\,,$$

$$\frac{\partial^2 f}{\partial x \partial y} = 6x\cos(x + 3y) - 3(x^2 + 2).\sin(x + 3y)$$

2. $a) -6, \quad b) -\dfrac{4}{25}, \quad c) -\dfrac{3}{4}, \quad d) \dfrac{4}{27}$

3. $a)\ x - 4y - z + 4 = 0, \qquad b)\ z = 0$

$c)\ x - y - z = 0 \qquad d)\ 3x + y - z - 1 = 0$

4. $a)\ \overrightarrow{grad}\, f(A) = (2, -1) \qquad b)\ \overrightarrow{grad}\, f(B) = (3,1)$

c) $\overrightarrow{grad}\, f(C) = \left(1, -\frac{\pi}{2}\right)$

5. $a)\ A = [0,0]$ lokální minimum

$b)\ B = [-1,2]$ lokální minimum

$c)\ C = [1,0]$ lokální minimum