Inquisitive Semantics II

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Questions express propositions

In inquisitive semantics questions are regarded as expressing a special kind of propositions.

The meaning of a sentence = its truth conditions

"To understand a proposition means to know what is the case if it is true."

L. Wittgenstein, TLP, 4.024

The sentential meaning of declarative sentences.

- In formal semantics, sentential meaning is usually identified with the informative content of the sentence.
- The informative content is modeled as a set of possible worlds.
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Truth-functional semantics for classical logic

A truth-functional model: $\mathcal{M} = \langle W, V \rangle$.

The relation of truth:

- p is true in w iff $w \in V(p)$,
- $\perp$ is not true in w,
- $\alpha \to \beta$ is true in w iff α is not true in w or β is true in w
- $\alpha \land \beta$ is true in w iff α is true in w and β is true in w

Propositions as sets of information states

- In inquisitive semantics, a proposition is not just a set of possible worlds but a set of sets of possible worlds (i.e. a set of information states).

Inquisitive semantics

An inquisitive model: $\mathcal{N} = \langle \mathcal{P}(W), V \rangle$.

The support relation:

$s \vDash p$ iff $s \subseteq V(p)$,

$s \vDash \bot$ iff $s = \emptyset$,

$s \vDash \varphi \to \psi$ iff for any $t \subseteq s$, if $t \vDash \varphi$ then $t \vDash \psi$,

$s \vDash \varphi \wedge \psi$ iff $s \vDash \varphi$ and $s \vDash \psi$,

$s \vDash \varphi \vee\!\!\!\vee\, \psi$ iff $s \vDash \varphi$ or $s \vDash \psi$.

Theorem

In every inquisitive model:

(a) *every formula is supported by the empty state,*

(b) *support is downward persistent for all formulas,*

(c) *support of declarative formulas is closed under arbitrary unions,*

(d) *every formula is equivalent to the inquisitive disjunction of a finite set of declarative formulas.*

Ontic and informational semantics

- As regards the declarative language the two semantics are equivalent:

  universal truth = universal support
  preservation of truth = preservation of support

- The standard framework is based on ontic objects (possible worlds) and an ontic relation of truth;
- The inquisitive framework is based on informational objects (information states = partial representations of possible worlds) and an informational relation of support.

Examples

a) Jane is in the cinema.
b) Is Peter in the cinema?
c) Is Jane in the cinema with Peter?
d) Peter or Jane is in the cinema.
e) Is Peter or Jane in the cinema?
f) Who is in the cinema: Peter or Jane?
g) If Peter is in the cinema, Jane is also there.
h) If Peter is in the cinema, is there also Jane?

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Intuitionistic logic

- $\alpha \to (\beta \to \alpha)$,
- $(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$,
- $(\alpha \wedge \beta) \to \alpha$,
- $(\alpha \wedge \beta) \to \beta$,
- $\alpha \to (\alpha \vee \beta)$,
- $\beta \to (\alpha \vee \beta)$,
- $(\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$,
- $\perp \to \alpha$.

- $\alpha, \alpha \to \beta / \beta$.

Basic Inquisitive Logic InqB

Intuitionistic logic plus

split $(\alpha \to (\psi \mathbin{\vee\!\!\!\vee} \chi)) \to ((\alpha \to \psi) \mathbin{\vee\!\!\!\vee} (\alpha \to \chi))$,

rdn $\neg\neg\alpha \to \alpha$,

where α ranges over $\mathbin{\vee\!\!\!\vee}$-free formulas.

Logic of problems

Kolmogorov, A. (1932). Zur Deutung der intuitionistischen Logik, *Mathematische Zeitschrift*, 35, 58–65.

- while classical logic captures the logical relations among statements, intuitionistic logic captures logical relations among problems

Medvedev logic of finite problems

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A remarkable result

Theorem

The schematic fragment of inquisitive logic corresponds to Medvedev logic of finite problems.

An example due to Ivano Ciardelli

- a certain disease may give rise to two symptoms: S_1, S_2
- hospital's protocol:

  > if a patient presents symptom S_2, the treatment is always prescribed; if the patient only presents symptom S_1, the treatment is prescribed just in case the patient is in good physical condition; if not, the risk associated with the treatment outweigh the benefits, and the treatment is not prescribed

A formalization of the protocol

The protocol:

- $t \leftrightarrow s_2 \vee (s_1 \wedge g)$

where

- s_1: the patient has symptom S_1
- s_2: the patient has symptom S_2
- g: the patient is in good physical condtion
- t: the treatment is prescribed

Types of information

Examples of types of information:

- patient's symptoms ($S_1, S_2, \ldots$)
- patient's conditions (good, bad)
- treatment (prescribed, not prescribed)

Types of information

Types of information correspond to questions:

- what are the patient's symptoms: $?s_1 \wedge ?s_2$
- whether the patient is in good physical conditions: $?g$
- whether the treatment is prescribed: $?t$

Dependencies among information types correspond to logical relations among questions

$$t \leftrightarrow s_2 \vee (s_1 \wedge g), ?s_1 \wedge ?s_2, ?g \vDash ?t$$

First-order inquisitive models

- in the first order setting: states are sets of first order structures
- truth conditions for universal quantifier and inquisitive existential quantifier:
  - $s \vDash \forall x \varphi[e]$ iff for every $a \in U$, $s \vDash \varphi[e(a/x)]$,
  - $s \vDash Ex\varphi[e]$ iff for some $a \in U$, $s \vDash \varphi[e(a/x)]$,

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Examples

Mention-all wh-questions

- Whom did Alice invite to her birthday party? $\forall x ?Pax$

Mention-some wh-questions

- What is a typical French dish? $ExFx$

The language $\mathcal{L}_{IEL}$

$$\varphi := \ p \mid \bot \mid \varphi \to \varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_a\varphi \mid E_a\varphi$$

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- $\neg\varphi =_{def} \varphi \to \bot$
- $\varphi \vee \psi =_{def} \neg(\neg\varphi \wedge \neg\psi)$
- $\varphi \leftrightarrow \psi =_{def} (\varphi \to \psi) \wedge (\psi \to \varphi)$
- $?\varphi =_{def} \varphi \Vvee \neg\varphi$
- $W_a\varphi =_{def} E_a\varphi \wedge \neg K_a\varphi$

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Epistemic modalities

the formula	represents
$K_a p$	The agent *a* knows that *p*.
$K_a?p$	The agent *a* knows whether *p*.
$E_a?p$	The agent *a* entertains whether *p*.
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- $K_a(\text{question}) = \text{statement}$
- $E_a(\text{question}) = \text{statement}$
- $W_a(\text{question}) = \text{statement}$

Declarative formulas

Definition

The set of declarative $\mathcal{L}_{IEL}$-formulas is the least set that contains all atomic formulas, $\perp$, $K_a\varphi$ and $E_a\varphi$, for any $\mathcal{L}_{IEL}$-formula φ, and is closed under $\wedge$ and $\rightarrow$.

Models

Definition

A concrete inquisitive epistemic model (CIE-model) is a triple $\langle W, \Sigma_{\mathcal{A}}, V \rangle$, where

- W is a nonempty set of possible worlds
- $\Sigma_{\mathcal{A}} = \{\Sigma_a \mid a \in \mathcal{A}\}$ is a set of inquisitive state maps
- V is a valuation assigning subsets of W to atomic formulas

Inquisitive state maps

- Σ_a assigns to every world w the issue of the agent a in the world w
- every issue is represented by a set of information states (those states that resolve the issue)
- every information state is represented by a set of possible worlds (those worlds that are compatible with the information, i.e. that are not excluded by the information)
- the information state of the agent in a world determines the boundaries for the issue of the agent in the world

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Inquisitive state maps

$\Sigma_a : W \to \mathcal{P}(\mathcal{P}(W))$, $\sigma_a : W \to \mathcal{P}(W)$ satisfying:

- $\Sigma_a(w)$ is nonempty downward closed,
- $\sigma_a(w) = \bigcup \Sigma_a(w)$,
- for any $w \in W$, $w \in \sigma_a(w)$ (factivity),
- for any $w, v \in W$, if $v \in \sigma_a(w)$, then $\Sigma_a(v) = \Sigma_a(w)$ (introspection).

Support conditions

- $s \vDash K_a\varphi$ iff $\forall w \in s$: $\sigma_a(w) \vDash \varphi$,
- $s \vDash E_a\varphi$ iff $\forall w \in s\ \forall t \in \Sigma_a(w)$: $t \vDash \varphi$.

Theorem

In every inquisitive epistemic model:

(a) *every formula is supported by the empty state,*

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Axiomatization of IEL

INT	Axioms of intuitionistic logic and modus ponens
split	$(\alpha \to (\varphi \mathbin{\vee\!\!\!\vee} \psi)) \to ((\alpha \to \varphi) \mathbin{\vee\!\!\!\vee} (\alpha \to \psi))$
rdn	$\neg\neg\alpha \to \alpha$

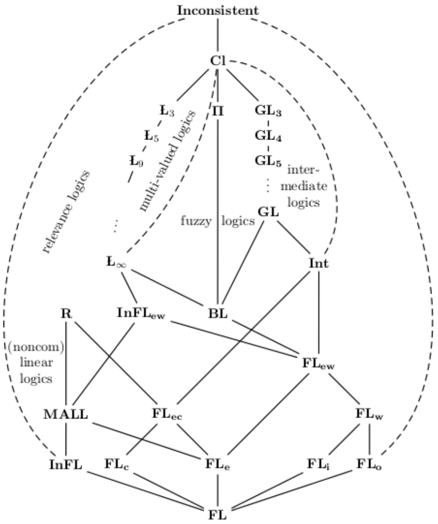
S5	S5-axioms and necessitation for K_a and E_a
K2	$K_a(\varphi \mathbin{\vee\!\!\!\vee} \psi) \leftrightarrow (K_a\varphi \vee K_a\psi)$
KE	$E_a\alpha \leftrightarrow K_a\alpha$

(α ranges over declarative formulas)

Is inquisitive logic a non-classical logic?

Two alternative approaches:

- inquisitive logic as a superintuitionistic logic in the standard propositional language
- inquisitive logic as a conservative extension of classical logic in an enriched language



Picture taken from Galatos, N. Jipsen, P. Kowalski, T., Ono, H. (2007) *Residuated Lattices: An Algebraic Glimpse at Substructural Logics*. Elsevier Science.