Intermediate macroeconomics
Dynamic macroeconomic model[1]

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IES - Summer Semester 2025

[1] Acknowledgment: These slides are partly based on slides of Tomas Lichard

Outline

Introduction

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Introduction

We have developed full model for understanding economic fluctuations.

- Key result: Real and nominal sides of the economy interact.
- Shocks cause fluctuations in output and prices (and other related variables).

Key problem: Absence of dynamics.

- Answers: AS-AD model provides a short-run comparative statics, but does explain how long will shocks affect the economy.
- Mechanisms: Modern macroeconomics see dynamic aspects as key driving mechanisms.

Solution: Dynamic macroeconomic model (DAS-DAD model).

- Dynamic models allow us to study how endogenous variables develop over time as they respond to shocks (impulse response functions).

Mechanisms: Aggregate demand foundations

What is the foundation behind aggregate demand?

- AD says demanded output is lower when prices are higher. But why?
- Higher prices decrease real money balances and hence increase interest rates and higher interest rates mean lower investment.

This is a good description of world with fixed money supply, e.g. gold standard.

- The model was developed in times of gold standard!

Problem: There is no such mechanism in modern economies with central banks setting interest rates!

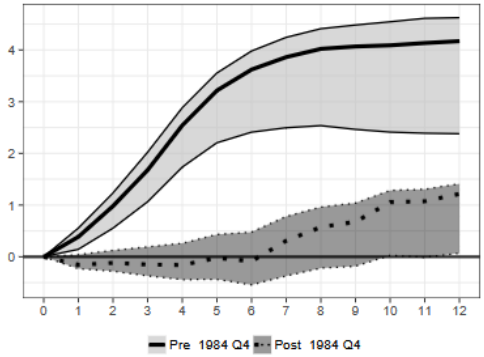
- Higher prices do not lead to higher interest rates *in absence of central bank action.*

We need alternative mechanism.

Mechanisms: Investment and interest rates

The AD/IS-LM model crucially relies on investment responsiveness to interest rates, but evidence suggests low responsiveness of investment to interest rates, especially in recent decades.

Change in borrowing costs that would prompt a change in investment plans	(1) In response to a decrease in interest rates	(2) In response to an increase in interest rates
	Number (pct.)	Number (pct.)
0.5 percentage point	17 (3%)	30 (6%)
1 percentage point	27 (5%)	52 (10%)
2 percentage points	43 (8%)	85 (16%)
3 percentage points	27 (5%)	61 (11%)
More than 3 percentage points	60 (11%)	108 (20%)
Likely would not change plans	367 (68%)	198 (37%)
Total	541 (100%)	534 (100%)
Memo: "Not applicable" answers	139	146



Mechanisms: Decisions about timing

Many economic decisions are decisions about timing.

- Consumption: Consume now or save and consumer more later.
- Investment: Investing lowers cash-flow now, but increases later.

Bad news: Standard IS-LM model does not reflect these important decisions at all.

Good news: This provides alternative way of linking demand and interest rates.

Mechanisms: Monetary policy

Monetary policy is a key factor in evolution of economy.

Monetary policy reacts to and influences the economy with lags.

- IS-LM model does not include these aspects (it assumes passive monetary policy).

We need model of monetary policy behavior.

Dynamic macroeconomic model

While AS-AD model is still useful for some contexts, the fact is that...

- Modern research does not use it at all.
- Policymakers mostly do not use it.
- Applied macroeconomists do not use it as their primary model.

We need an alternative, dynamic macroecnomic model.

- Good news: Most key aspects are shared with AD-AS.

Model structure

Outline

Basic blocks

Also called IS-PC-MPR model, because it consists of:

1. IS curve
2. Phillips curve
3. Monetary policy rule

Demand side: IS curve

Assume that demand for goods and services is given by

$$y_t = \overline{y}_t - \alpha(r_t - \rho) + \epsilon_t$$

- y_t is log of total output.
- $\overline{y}_t$ is log of the natural level of output.
- r_t is the real interest rate.
- $\rho > 0$ is a **natural rate of interest**.
- $\alpha > 0 \Leftrightarrow$output is lower when real interest rate is higher.
- ϵ_t is a demand shock (e.g., sentiment, government spending).

Demand side: Microfoundations

The demand equation says that demand is related to *real* interest rate. Where does this come from?

This can be microfounded in several ways.

- Simplest: consumption and inter-temporal substitution.
- Alternative: Investment and cost of external financing.

Supply side: Phillips curve

Supply side takes the form of Phillips curve:

$$\pi_t = \pi_t^e + \phi(y_t - \overline{y}_t) + \nu_t$$

- $y_t - \overline{y}_t$: output gap
- $\phi > 0$: sensitivity of inflation to output fluctuations
- ν_t: supply shock

The supply equation says that inflation is related to output. Where does this come from?

Microfoundations: Price stickiness, wage stickiness or imperfect information.

Monetary policy rule

Assume that monetary policy reacts to excess inflation and output:

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \overline{y}_t)$$

- π_t^* is the inflation target
- $\theta_\pi > 0$, $\theta_y > 0$: policy parameters

Interpretation: Central bank has inflation target and dislikes recessions.

Typically called the **Taylor rule**.

- Based on observed historical behavior of interest rates.

The monetary policy rule: Parametrization

Monetary policy rate depends on 4 variables (actual and target inflation, actual and potential output) and 2 parameters.

- θ_π controls how much does central bank react to inflation.
- θ_y controls how much does central bank react to output.

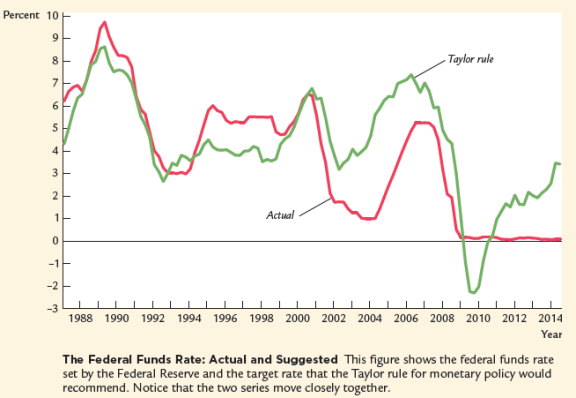
Assumption $\theta_\pi > 0$ ensures that interest rates move more than inflation $\Leftrightarrow$real interest increase/decrease in response increase/decrease in inflation.

- Stabilization property - pushing inflation back to target (see later).
- Called the Taylor principle.

The monetary policy rule: Empirics

Before Great recession Taylor rule provided a relatively good fit for actual policy.

- After Great recession policy was constrained by zero-lower bound.



The Federal Funds Rate: Actual and Suggested This figure shows the federal funds rate set by the Federal Reserve and the target rate that the Taylor rule for monetary policy would recommend. Notice that the two series move closely together.

The Fisher equation

Define real interest rate according to the Fisher equation:

$$r_t = i_t - E_t \pi_{t+1}$$

This is ex-ante real interest rate; depends on expected inflation.

Expected inflation

Assume adaptive (purely backward-looking) expectations.

$$E_t \pi_{t+1} = \pi_t$$

Alternatives:

- Anchored inflation expectations: $\pi_t - \pi^* = \psi(\pi_{t-1} - \pi^*)$
- Rational/Model consistent expectations: Optimal expectations given information and model.

Summary of model equations

Three main + two auxiliary equations:

1. IS: $y_t = \overline{y}_t - \alpha(r_t - \rho) + \epsilon_t$
2. PC: $\pi_t = \pi_{t-1} + \phi(y_t - \overline{y}_t) + \nu_t$
3. MR: $i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \overline{y}_t)$
4. Fisher: $r_t = i_t - E_t\pi_{t+1}$
5. Inflation expectations: $E_t\pi_{t+1} = \pi_t$

Model variables

Purpose of macroeconomic models is to determine current and future values of model variables.

There are three types of variables in any economic model:

- Endogenous variables: Variables that are determined by the model.
- Exogenous variables: Variables that enter the model, but are not determined by it.
- Pre-determined endogenous variables: Variables that are endogenous in principle, from current perspective are already determined.

Model variables (continued)

Our model includes 10 different variables:

Endogenous Variables	
Y_t	Output
π_t	Inflation
r_t	Real interest rate
i_t	Nominal interest rate
$E_t\pi_{t+1}$	Expected inflation
Exogenous Variables	
$\overline{Y}_t$	Natural level of output
π_t^*	Central bank's target for inflation
ϵ_t	Shock to the demand for goods and services
υ_t	Shock to the Phillips curve (supply shock)
Predetermined Variable	
π_{t-1}	Previous period's inflation

Model parameters

Apart from variables, the model equations also include parameters.

- Parameter is essentially a number, just one that is not fixed, but one that can (in principle) be changed.
- Contrast with variables: Variables change as result of shocks, changes in parameters are shocks.

Model parameters (continued)

Our model has 5 parameters.

Parameters	
α	The responsiveness of the demand for goods and services to the real interest rate
ρ	The natural rate of interest
ϕ	The responsiveness of inflation to output in the Phillips curve
θ_π	The responsiveness of the nominal interest rate to inflation in the monetary-policy rule
θ_Y	The responsiveness of the nominal interest rate to output in the monetary-policy rule

Model equilibrium

Outline

Introduction

Once we have our model structure defined we want to use the model for analysis of the behavior of model variables.

A macroeconomic model has two types of equilibria:

- Long-run equilibrium.
- Short-run equilibrium.

Setting model parameters

To use the model we need to specify the model parameter values.

How do we determine parameters?

- Option 1: Estimate them.
  - Finding parameter values that lead to best model fit.
- Option 2: Calibrate them.
  - Find parameter values that allow the model to match some salient features of data.

Even simple models are highly non-linear, so hard to estimate *precisely*.

- In 1980s and 1990s models were calibrated.
- Since mid 2000s models are often estimated using Bayesian methods.

Model parameter values

We will use following model parameter values:

- $\alpha = 1$: Real rate moves output 1-for-1
- $\rho = 2$
- $\phi = 0.25$: Inflation increases 0.25p.p. for 1p.p increase in output gap.
- $\theta_y = 0.5$ and $\theta_\pi = 0.5$: *Real* rates react equally to inflation and output gap.

In addition we will normalize $\overline{y} = 100$ and use $\pi_t^e = 2$ corresponding to common inflation target.

Long-run equilibrium

First step in analyzing model: Defining and investigating the long-run equilibrium.

- In more complex models we also need to worry whether any stable long-run equilibrium exists.

How should we define long-run equilibrium in our model? $\Rightarrow$ everything, including inflation, is stable.

- $y_t = y_{t-1}$, $\pi_t = \pi_{t-1}$,$i_t = i_{t-1}$, $r_t = r_{t-1}$.

Logically $y_t = \overline{y}_t$ and $\pi_t = \pi^e_{t+1} = \pi^*_t$.

- Equilibrium output is at potential, inflation is at target.

For this to hold we will need $r_t = \rho$ and hence $i_t = \rho + \pi^*_t$.

- Real rate equals to natural rate.
- $\rho + \pi^*_t$ is the long-run neutral rate of nominal interest rates.

Short-run equilibrum

We have 5 equations/endogenous variables. This is impossible to analayze effectively.

- Moreover, our equations are simultaneous: To know output you need to know real interest rate, for which you need to know nominal interest rate, for which you need to know output...

Two solutions:

- Simplify into 2 variable space so that we can show most relevant aspects on single (2-dimensional) graph.
- Simulate the responses of the model to shocks and analyze multiple graphs.

We will start with first approach, and then proceed with the second one.

Dynamic AS curve

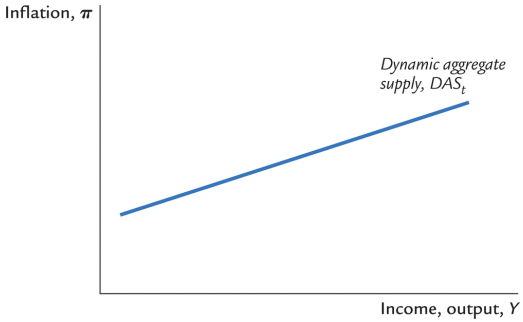
Combining PC with inflation expectations equation yields dynamic aggregate supply curve:

$$\pi_t = \pi_{t-1} + \phi(y_t - \overline{y}_t) + \nu_t$$

DAS curve illustration

DAS is upward sloping because of Phillips-curve relationship: More output means higher prices/inflation (demand-pull mechanism).

- Change in output causes movement along DAS curve, changes previous inflation, potential output or supply shock move the DAS curve.



[S] Derivation of the DAD Curve

We will use the remaining equation to derive the dynamic aggregate demand curve:

$$\begin{aligned}
\textbf{IS:} \quad & y_t = \bar{y}_t \; - \; \alpha\left(r_t - \rho\right) \; + \; \epsilon_t, \\
\textbf{Fisher:} \quad & r_t = i_t - E_t\pi_{t+1}, \\
\textbf{Monetary Rule:} \quad & i_t = \pi_t + \rho \; + \; \theta_\pi\left(\pi_t - \pi_t^*\right) \; + \; \theta_y\left(y_t - \bar{y}_t\right), \\
\textbf{Expectations:} \quad & E_t\pi_{t+1} = \pi_t.
\end{aligned}$$

Substitute the Fisher equation into the IS curve:

$$y_t \; = \; \bar{y}_t \; - \; \alpha\Big(i_t - E_t\pi_{t+1} - \rho\Big) \; + \; \epsilon_t.$$

Substitute in the monetary-policy rule for i_t:

$$y_t = \bar{y}_t - \alpha\Big[\pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \bar{y}_t) - E_t\pi_{t+1} - \rho\Big] + \epsilon_t.$$

[S] Derivation of the DAD Curve (continued)

Use $E_t\pi_{t+1} = \pi_t$ and cancel the ρ– and π_t–terms:

$$y_t = \bar{y}_t - \alpha\left[\theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \bar{y}_t)\right] + \epsilon_t.$$

Collect like terms in y_t:

$$(1 + \alpha\theta_y)\, y_t = (1 + \alpha\theta_y)\, \bar{y}_t - \alpha\, \theta_\pi\, (\pi_t - \pi_t^*) + \epsilon_t.$$

Solve for y_t to obtain the DAD curve:

$$\boxed{y_t = \bar{y}_t - \frac{\alpha\, \theta_\pi}{1 + \alpha\, \theta_y}\, (\pi_t - \pi_t^*) + \frac{1}{1 + \alpha\, \theta_y}\, \epsilon_t}$$

DAD curve

The final result is the dynamic aggregate demand curve which relates output to inflation and other variables:

$$y_t = \overline{y}_t - \left[\frac{\alpha\theta_\pi}{1+\alpha\theta_y}\right](\pi_t - \pi_t^*) + \left[\frac{1}{1+\alpha\theta_y}\right]\epsilon_t$$

or more concisely:

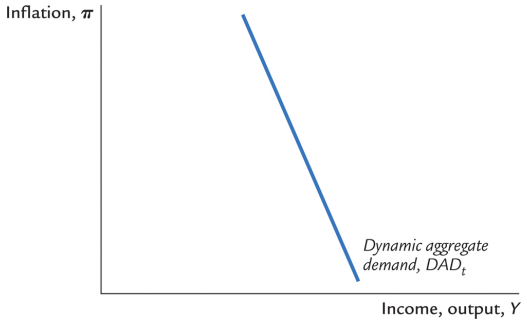
$$y_t = \overline{y}_t - \gamma_1(\pi_t - \pi_t^*) + \gamma_2\epsilon_t$$

Note: Assumes monetary policy follows the MPR *exactly* and expectations are *exactly* backward looking.

DAD curve illustration

DAD curve reflects negative relationship between inflation and demand for goods and services.

- Change in inflation causes movement along DAD curve, changes in potential output, inflation target or demand shock move the DAD curve.



Slope of DAD curve

Why is DAD downward sloping?

- CB reacts to higher inflation by increasing nominal interest rates more than inflation, causing real interest rates to rise, lowering demand for goods and services.
- What if nominal interest rates rise less than inflation? $\rightarrow$ real interest rates fall, stimulating demand, ... $\Rightarrow$ DAD would become upward sloping.

What determines slope of DAD? $\rightarrow$ The reaction function of CB.

- Higher θ_π relative to θ_y means more steep DAD.

DAD vs AD

DAD and AD seem similar - only inflation instead of prices.

But they reflect very different mechanisms:

- AD: Increase in prices increases interest rates due to decline in real monetay balances.
  - No role of central bank.
- DAD: CB increases real interest rates in reaction to inflation.
  - Crucially relies on interest rates.

Short-run equilibrium

The dynamic, short-run equilibrium is determined by DAD-DAS curves, which together (simultaneously) determine output and inflation

$$y_t = \overline{y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_y}(\pi_t - \pi_t^*) + \frac{1}{1+\alpha\theta_y}\epsilon_t$$

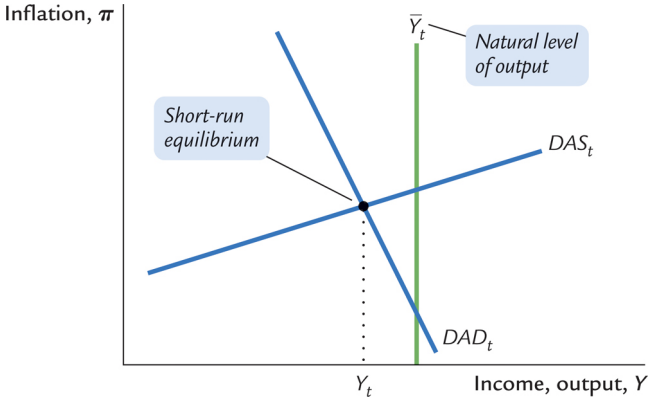
$$\pi_t = \pi_{t-1} + \phi(y_t - \overline{y}_t) + \nu_t$$

Note that while DAS is dynamic (links across time), DAD is not.

- After initial effect of shocks only DAS will be moving, DAD will remain unchanged.

Short-run equilibrium: Illustration

Short-run equilibrium is given by intersection of DAD and DAS, and can have output above or below natural level, and inflation above or below target.



[A] Short-run equilibrium: Algebraic solution

One can also solve for the equilibrium algebraically, obtaining endogenous variables are functions of shocks.

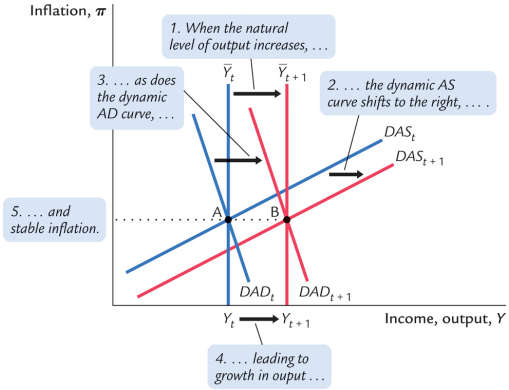
$$Y_t = \overline{Y}_t + \frac{\alpha\,\theta_{\pi t}\left(\pi_t^* - \pi_{t-1} - \nu_t\right) + \epsilon_t}{1 + \alpha\left(\theta_{\pi t}\,\phi + \theta_{Y_t}\right)},$$

$$\pi_t = \frac{\left(1 + \alpha\,\theta_{Y_t}\right)\left(\pi_{t-1} + \nu_t\right) + \alpha\,\theta_{\pi t}\,\phi\,\pi_t^* + \phi\,\epsilon_t}{1 + \alpha\left(\theta_{\pi t}\,\phi + \theta_{Y_t}\right)}.$$

These can then be plugged into the remaining equations to obtain solution for i_t, π_{t+1}^e and r_t.

Increase in potential output

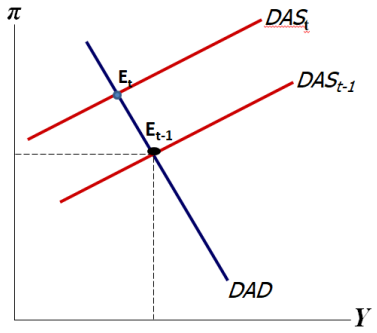
Increase in potential causes same movement in DAD and DAS leading to higher output and unchanged inflation.



Shock to DAS

Negative supply shock shifts DAS up, causing lower output and higher inflation.

- Why is output lower? $\rightarrow$ CB increases nominal/real interest rate.
- Note that inflation increases less than supply shock due to immediate CB reaction.



Shock response simulations

Outline

Introduction

The DAD-DAS diagram can be used to analyze effects of shocks, but it has few limitations when it comes to analyzing further dynamic behavior.

- It can become very clunky when we also capture dynamics.
- It shows the effects only on output and inflation, with movements of other variables invisible.
- It does not capture the source of dynamic adjustments.

For analyzing dynamic behavior of our model it is more suitable to look at plots of responses to shocks.

- We will now look at large number of shock responses to understand the model.

Supply shocks

Supply shock

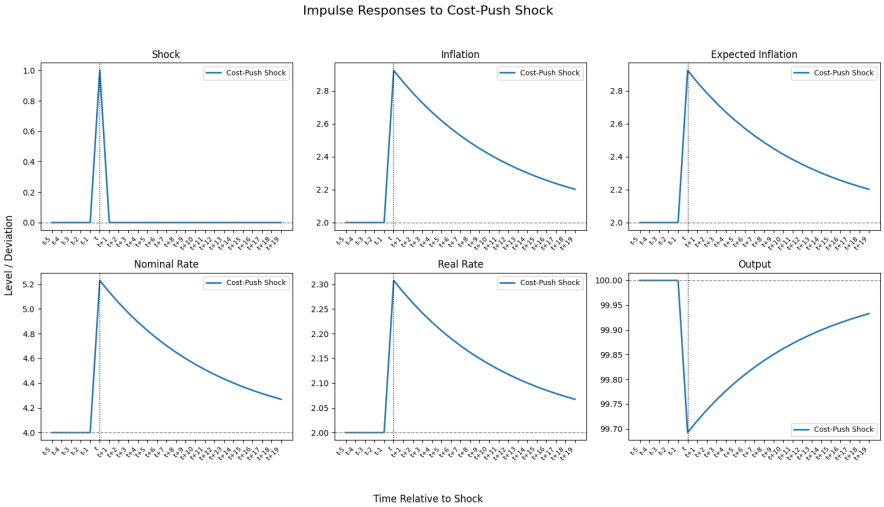
We will now consider a effect of supply shock.

What are some examples of supply shocks?

- Commodity price shock.
- Wage shock.
- Financial costs shocks (e.g. financial crisis).
- ...

  $\Rightarrow$ Anything that increases inflation without *directly* increasing output.

Supply shock dynamic response



Supply shock dynamic response: Mechanism I

We saw in DAD-DAS diagram that supply shock raises inflation and lowers output.

- The mechanism behind lower output is increase in i that exceeds increase increase in π_t and π_t^e so that r_t is higher.
- Note that inflation increased less than the size of supply shock.

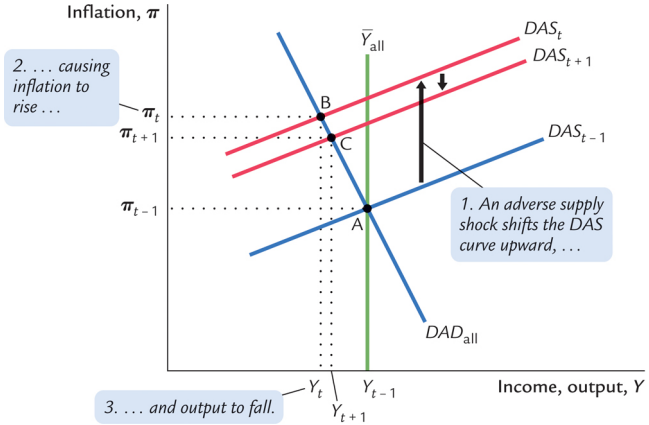
Supply shock dynamic response: Mechanism II

What happens after initial shock?

- Inflation is still above target because of higher expected inflation but it is lower than before because ν_t is now 0 and inflation in previous period increased less than the shock ($\pi_{t-1} < \nu_{t-1}$). $[\pi_t = \pi_{t-1} + \phi(y_t - \overline{y}_t) + \nu_t]$
- Inflation still above target means that i_t above natural level, but less than before because inflation is lower.
- r_t also decreases because i_t decreases more than inflation (see MPR equation), but stays above natural level. $[i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \overline{y}_t)]$
- Lower r_t means higher output. $[y_t = \overline{y}_t - \alpha(r_t - \rho) + \epsilon_t)]$
- Lower inflation means that future inflation will be lower through effect on expected inflation/Phillips curve equation.
- ...

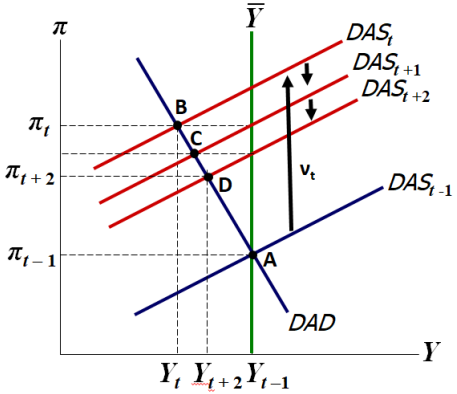
Shock to DAS: Dynamics

In future periods DAS will continue to move as previous inflation changes, while DAD remains unchagned (we move along DAD).



Shock to DAS: Dynamics

The movement continues until economy reaches long-run equilibrium with inflation at target/nominal interest rates at neutral level/real interest rates at natural level.



Supply shock dynamic response: Role of monetary policy

Simplified narrative: In response to supply shock central bank increases interest rates, causing output to drop below natural level, to which firms react by lowering their prices due to weak demand what causes inflation to drop to which central bank reacts by lowering interest rates.

The restrictive monetary policy causes recession. Why does CB do that?

- CB implements restrictive monetary policy to ensure that inflation converge back to target.

How fast do we return to long-run equilibrium?

- Could central bank ensure inflation never spikes? Yes! Just increase i_t enough! ($\frac{\nu_t}{\phi} = 4$)
- The problem: Very severe recession.

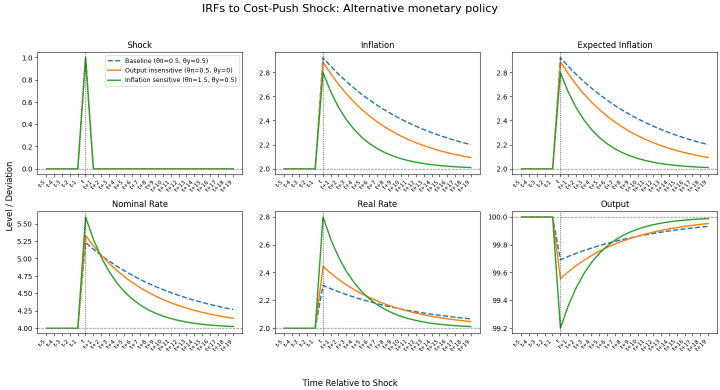
  $\Rightarrow$ CBs face trade-off between stable inflation and recession, and navigate this trade off.

  $\Rightarrow$ The speed of return is determined by CB policy.

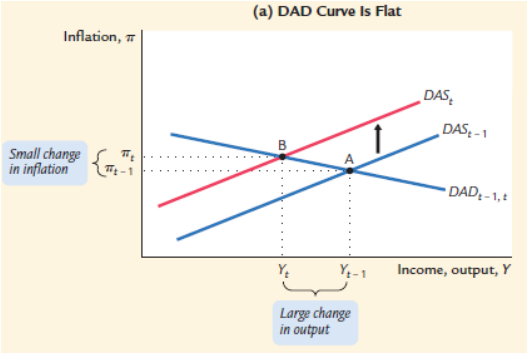
Alternative monetary policy I

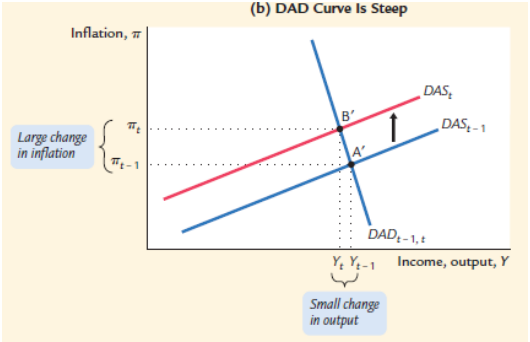
Higher coefficient θ_π and/or lower θ_y means faster return to inflation target but at cost of bigger recession.

- While recession is shorter the sacrifice ratio is higher.

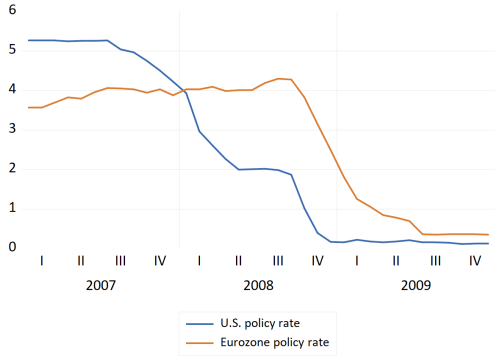


Alternative monetary policy I: Diagram



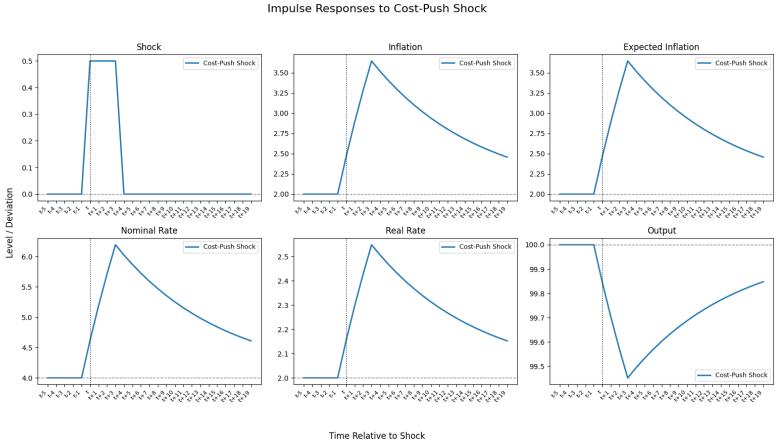


Alternative monetary policy I: Great recession



Supply shock dynamic response II

Inflation starts dropping immediately after one period supply shock → multi-period shock means inflation continues to increase.



Alternative monetary policy II

The MPR with $\theta > 0$ implies i_t increases more than π_t, so that r_t increases and hence output decreases, stabilizing inflation through demand-pull mechanism.

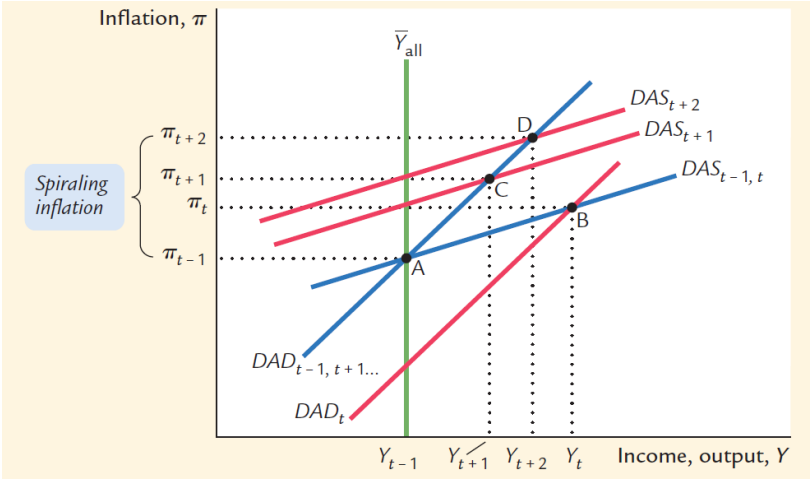
What would happen if $\theta < 0$?

- i_t increase, but less than π_t.
- As a result r_t declines...
- ...causing output to increase...
- ...what leads to increase in inflation due to demand-pull mechanism.
- ...

Alternative monetary policy II: Dynamic response

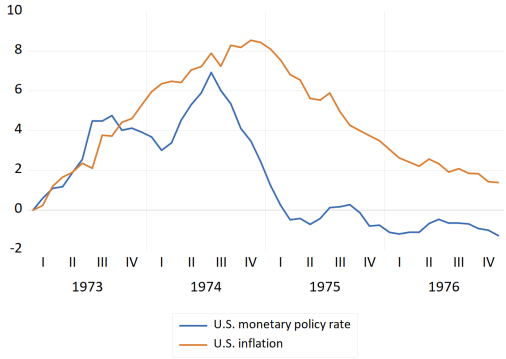
Alternative monetary policy II: Diagram

If $\theta_\pi < 0$ then DAD is upward sloping and the macroeconomy becomes unstable.



Alternative monetary policy II: Great inflation

During Great inflation in 1970s central bank increased interest rates less than one-for-one with inflation.



Alternative expectations formation

The reason why central bank responds with restrictive policy to supply shock is because otherwise inflation expectations de-anchor.

- This crucially depends on the backward inflation expectations formation.

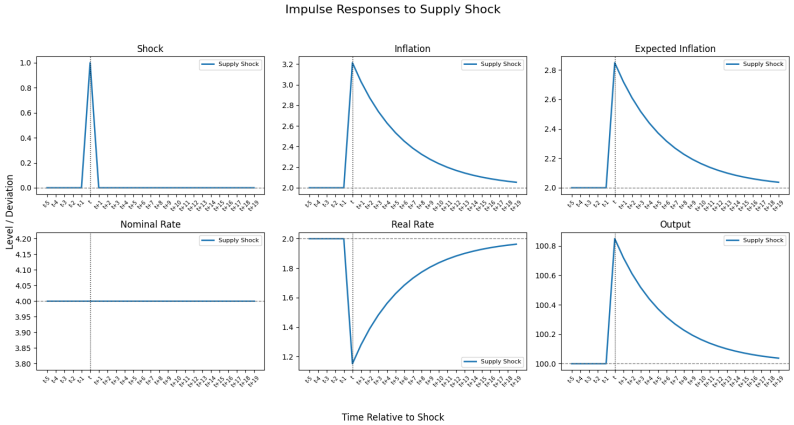
What if inflation expectations are anchored? $[\pi_t - \pi^* = 0.7(\pi_{t-1} - \pi^*)]$

- Expectations increase less than inflation itself.

Alternative expectations formation: Illustration

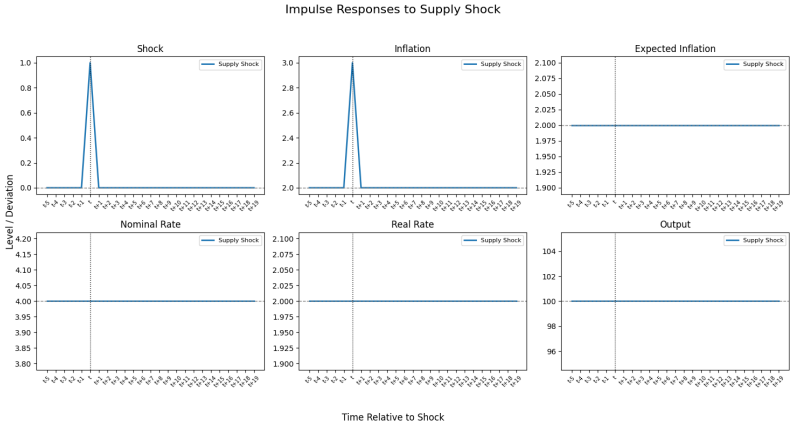
Inflation converges back to target despite no reaction from central bank.

- As long as demand channel is not strong enough.



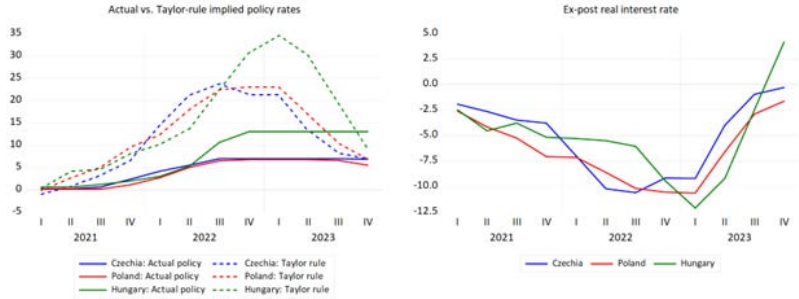
Alternative expectations formation: Illustration II

If expectations are fully anchored (do not react to inflation) then does not react at all.



Alternative expectations formation: Post-pandemic inflation

After pandemic inflation return to targets despite central banks raising rates much less than Taylor rule ⇔(ex-post) real interest rates being deeply negative.



Demand shocks

Demand shock

We will now consider a effect of demand shock.

What are some examples of demand shocks?

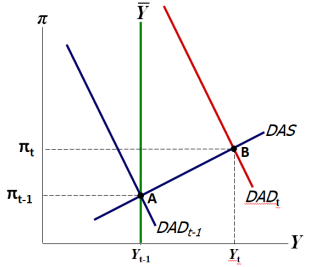
- Fiscal policy.
- Confidence shock.
- Increase in wealth.
- Lower demand from abroad.
- ...

  $\Rightarrow$ Anything that increases output without *directly* increasing inflation.

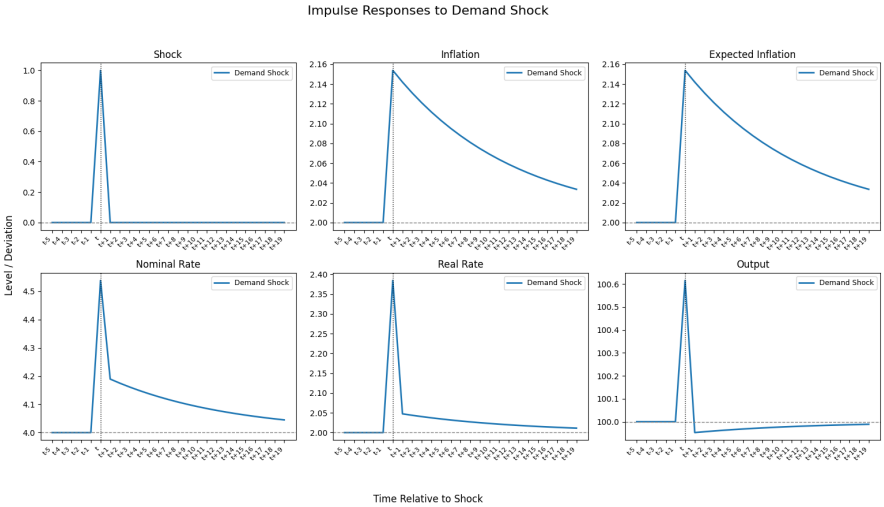
Demand shock response

Demand shock leads to increase in output and to (indirect) increase in inflation due to demand-pull mechanism.

- The increase in output is smaller than the shock because of reaction by central bank to inflation.



Demand shock dynamic response



Demand shock dynamic response: Mechanism I

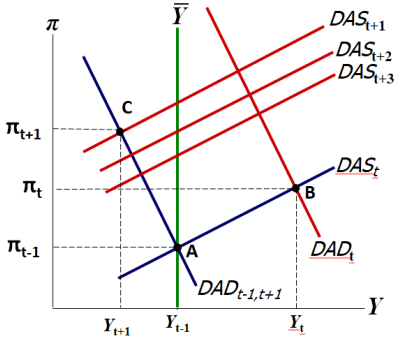
What happens after initial shock?

- End of demand shock means decline in output. $[y_t = \overline{y}_t - \alpha(r_t - \rho) + \epsilon_t)]$
- Increase in inflation in previous period leads to increase in expected inflation in previous period.
- This causes increase in future inflation (shift in DAS). $[\pi_t = \pi_{t-1} + \phi(y_t - \overline{y}_t) + \nu_t]$
- Overall, inflation is above target despite output returning to normal.
- This means that central bank sets i_t restrictive territory, so that r_t is above natural level. $[i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \overline{y}_t)]$
- What means that output is now below natural level.
- Output below neutral means inflation is pushed lower (demand pull in reverse).
- Lower inflation means lower expected inflation (shift in DAS).
- ...

[S] Demand shock dynamic response: Diagram

Demand shock leads to increase in output and to (indirect) increase in inflation due to demand-pull mechanism.

- The increase in output is smaller than the shock because of reaction by central bank to inflation.



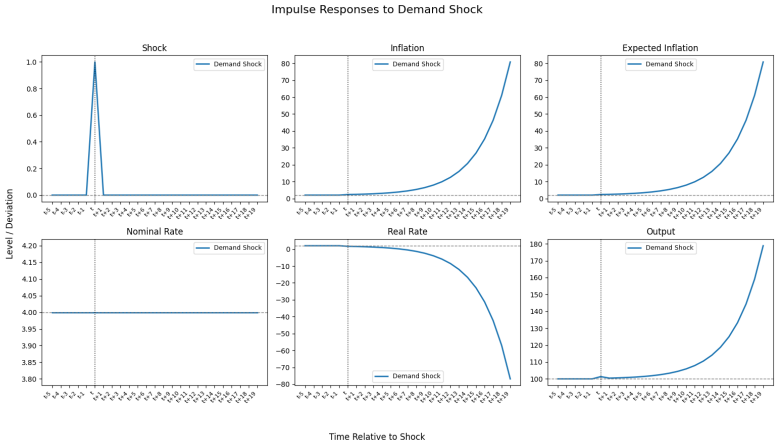
Behavior of monetary policy

Again the behavior of monetary policy is crucial in the responses of shocks.

- In the period of shock i_t is increased because $y_t > \overline{y}$ and $\pi_t > \pi^* \Rightarrow$ CB is fighting overheating economy with policy restriction.
- In the following period i_t decreases as economy is no longer overheating $y_t < \overline{y}$...
- ...but it does not decrease back to neutral rate because inflation is still above target $\Rightarrow$ CB is fighting de-anchored inflation expectations with policy restriction (i.e. recession).

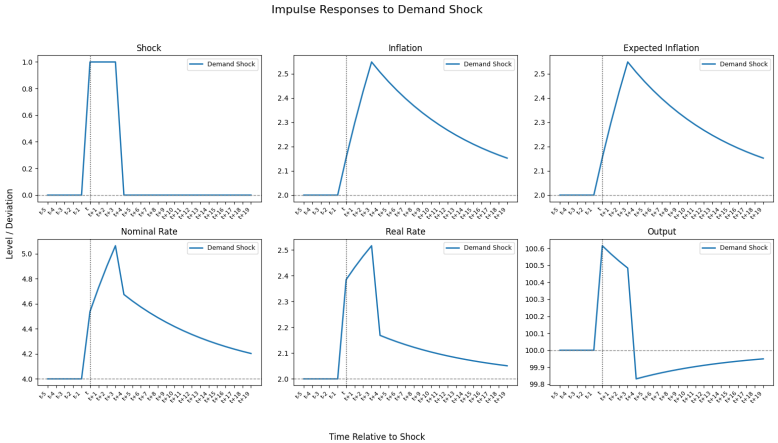
Behavior of monetary policy II

What if CB does not react? Inflation expectations de-anchor further, leading the economy into inflationary spiral.

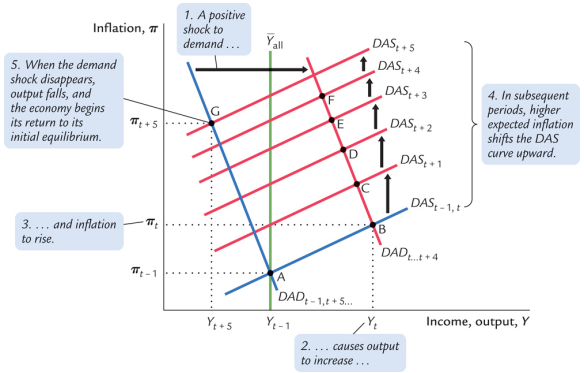


[S] Multiple period shock

Inflation starts dropping immediately after one period demand shock → multi-period shock means inflation and interest rates continues to increase, while output declines.

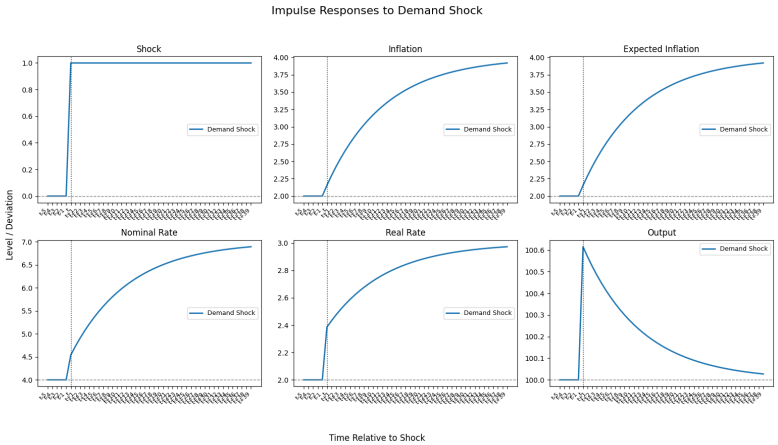


[S] Multiple period shock: Diagram



[S] Permanent shock

Permanent shock would result in higher inflation, nominal interest rate, real interest rate but the same output.



Monetary policy shocks

Monetary policy shocks

We will now consider a effect of monetary policy shock.

What are some examples of monetary policy shocks?

- Change in the policy rule specification.
- Change in inflation target.
- Change in interest rate unjustified by inflation or output movements.
- Change in natural rate ρ not matched by central bank.

  $\Rightarrow$ Any change in the MPR equation.

Monetary policy rate shock

We will first consider one time change in policy rates $\Leftrightarrow$ policy rule remains unchanged.

We need to introduce policy shock into the equation

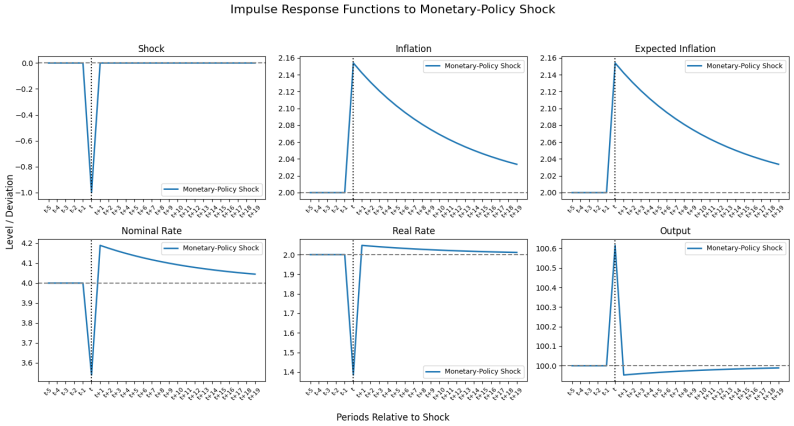
$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_y(y_t - \overline{y}_t) + \eta_t$$

where η_t is a policy shock $\Leftrightarrow$ *unexpected* deviation from the policy rule.

- Systematic Behavior of central bank remains unchanged.

Policy rate shock

Assume one-period shock to η_t, after which $\eta_t = 0$.



Policy rate shock: Interpretation

The policy shock causes chain of events:

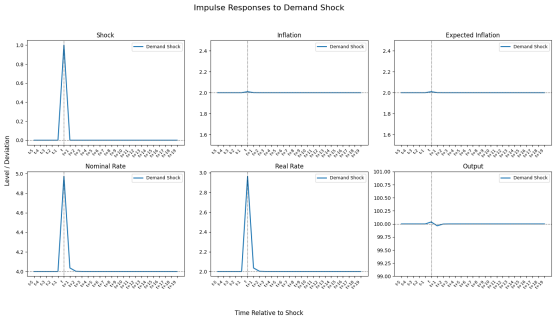
- Nominal interest rate drops, leading to decrease in real interest rate.
- Lower real interest rate means higher output and inflation.
- Higher output and inflation lead CB to *immediately* increase rates (i.e. fight against its own shock).
- Higher current inflation leads to higher expected inflation.
- Following period inflation is higher due to higher expected inflation.
- Central bank reacts by increasing the nominal interest rate above neutral in order to cool the economy so that inflation decreases.
- Output decreases below natural level, gradually pushing inflation lower.
- As inflation decreases nominal interest rates return to normal.

Policy rate shock: Interpretation II

Monetary policy rate shock is equivalent demand shock!

- Causes identical reaction in the macro variables apart from nominal interest rate.

Indeed, demand shocks can be considered monetary policy shocks, because CB could always offset demand shock *completely* be increasing interest rate.



Different policy shocks

Policy rate shock assumes that central bank makes mistake of commission: It changes policy rate when it should not.

Central bank can also make mistake of omission: Not change policy rates when it should.

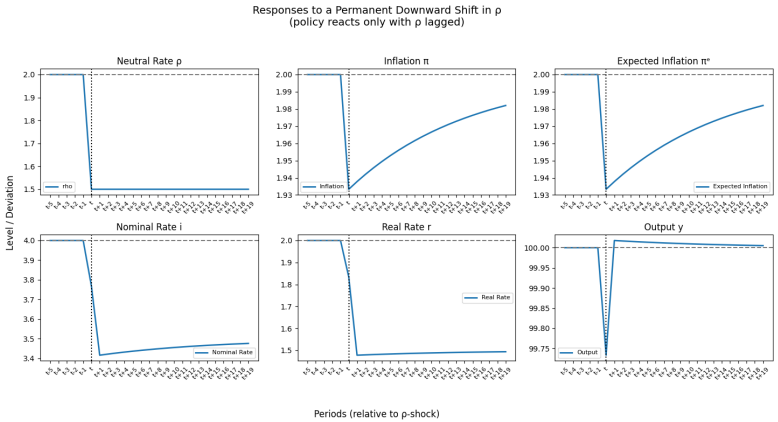
- Demand or supply shock calls for change in rates, but CB does not react $\rightarrow$ aggravating the shocks.

  $\Rightarrow$Same implications as mistake of commission.

Particular interesting case: Permanent change in natural interest rate.

[S] Natural interest rate shock

Assume that the natural interest rate unexpectedly declines and CB realizes that only with delay.



[S] Natural rate shock: Interpretation

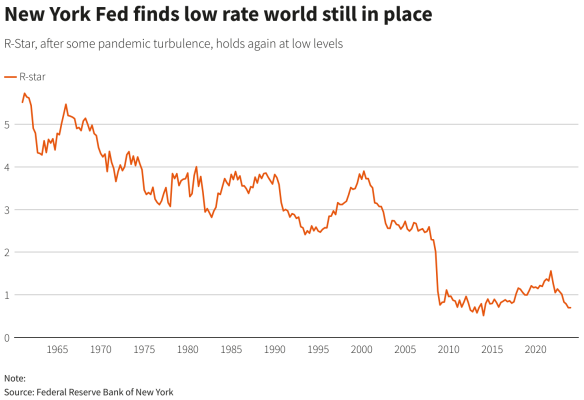
In the period of decline in natural rate the real interest rate is above natural rate, causing...

- Decrease in output. $[y_t = \overline{y}_t - \alpha(r_t - \rho) + \epsilon_t]$
- Decrease in inflation and expected inflation.

  $\Rightarrow$Similar to positive policy rate shock or negative demand shock.
- Afterwards policy rate reacts, leading to decrease in nominal rate, which overshoots the neutral rate in order to stimulate the economy and hence increase inflation
- In the long run inflation and output are unchanged, but nominal and real interest rates are lower.

Natural interest rate shock: Empirics

There is a substantial evidence that natural interest rate has decreased over time.



[S] Natural rate shock: Potential sources

There are several potential reasons for natural interest rate to decline:

- Demographics: aging populations and slower population growth can mean higher household savings & lower investment
- Productivity: slower productivity growth means lower return on investment.
- International environment: excess savings in foreign countries will drag down domestic interest rates (Global Savings Glut).
- Secular stagnation: chronically weak business investment demand

Inflation target shock

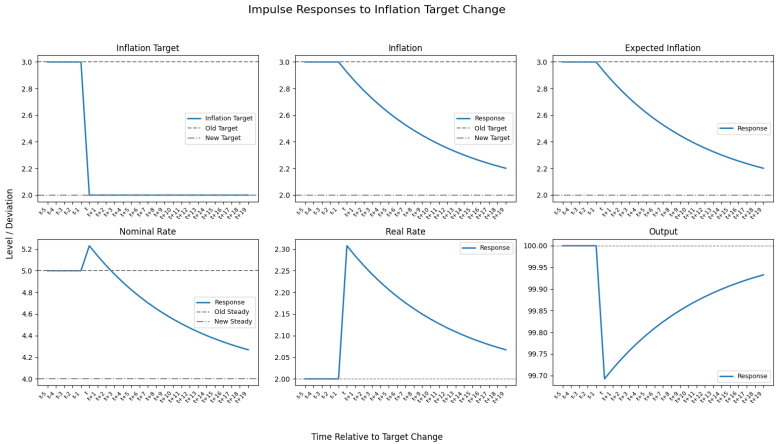
Another possible policy shock is change in inflation target.

Typical context: After period of high inflation central bank wants to set inflation at lower level.

- Disinflation policy ala Volcker.

Dynamic macroeocnomic model allows us to study the mechanisms of disinflation.

Inflation target shock



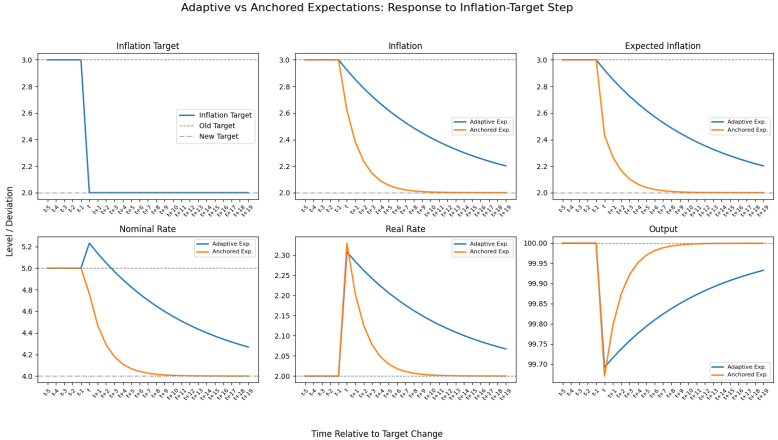
Inflation target shock: Interpretation

To lower inflation the central bank need to rely on demand pull inflation: engineer recession to pull inflation down.

- Nominal interest rates are increased, while expected inflation remains unchanged, causing real interest rate to rise.
- Higher real interest rates mean lower output and hence lower inflation.
- Lower inflation means lower expected inflation.
- Lower expected inflation and still high interest rates mean lower future inflation.
- Eventually nominal interest rates end up lower, while real interest rate converge back to natural level.

Inflation target shock: Shift in expectations

The disinflation is much less costly is expectations are not completely backward looking $[\pi_t - \pi^* = 0.7(\pi_{t-1} - \pi^*)]$.



Conclusions

Outline

New model

We have developed dynamic macroeconomic model:

- Some stuff like before: Supply side, Fisher equation, inflation expectations.
- Some stuff changed: Demand side.
- Some stuff is new: Monetary policy rule.

The changes are fundamental.

- Active rather than passive monetary policy.
- Completely different mechanism for relationship between prices and output.

Dynamic nature of the model

Main difference: This is an explicitly dynamic model.

- AS-AD allowed for comparative static exercises following shock.
- AS-AD lacked fully specified dynamic process for expectations formation, so did allow for dynamic exercises.

Lessons

The behavior of economy crucially depends on the behavior of the central bank.

- Following supply shock central bank faces trade-off: It needs to hurt the economy to bring inflation back to target.
- Following demand shock central bank cools down the economy to bring inflation back to target.
- Monetary policy interest rate shocks are like demand shocks.

If central bank reacts more to shocks then economy returns to long-run equilibrium faster, but at more cost (e.g. bigger recession following a supply shock).

The other crucial aspect affecting profile of responses to shocks is the expectations formation.

What's next?

Advance courses will focus on microfoundations of dynamic macro model.

- Consumer optimization and derivation of IS curve.
- Firm optimization and derivation of PC curve.
- Taylor rule as optimal policy rule based on welfare considerations.

Mircofoundations provide some nuance, but do not change the big picture.

Modern research

- **Heterogeneous-Agent Dynamics**: Incorporating income, wealth and firm-level dispersion into DSGE frameworks; understanding distributional effects of shocks and policy.
- **Financial Frictions & Macro–Finance Linkages**: Endogenizing credit constraints, collateral, and banking sectors; assessing systemic risk and macroprudential policies.
- **Expectation Formation & Learning**: Modeling boundedly rational agents and adaptive expectations; exploring the role of "news shocks" in business-cycle fluctuations.
- **Network Macro & Spillover Effects**: Capturing supply-chain and financial-network interdependencies; quantifying shock transmission across firms, sectors, and countries.
- **Inequality, Demography & Labor-Market Frictions**: Linking demographic change and labor-force heterogeneity to aggregate dynamics; studying fiscal and social-insurance policies in dynamic general equilibrium.
- **Climate Change & Environmental Shocks**: Integrating physical climate risks into growth and business-cycle models; valuing adaptation policies and "green" fiscal interventions.